Climate Change and Long-Horizon Portfolio Choice: Combining Theory and Empirics*

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Abstract

We propose a novel approach for measuring the impact of climate change on long-horizon equity risk and optimal portfolio choice. Our method combines historical data about the impact of climate change on return dynamics with prior beliefs elicited from the temperature long-run risk (LRR-T) model of Bansal, Kiku, and Ochoa (2019). Our Bayesian framework incorporates this prior information to obtain more precise estimates of long-term climate risks than existing methods that solely rely on historical data. Compared to the benchmark investor without climate change, we document that the LRR-T Bayesian investor predicts higher equity premia for all investment horizons, with per period variance increasing considerably over the horizon. This results in relatively (high) low allocations to equities in the (short) long run. Investors that optimize between portfolios that are vulnerable and non-vulnerable to climate change only diversify in the long run.

Keywords: climate change, long-run risk, prior beliefs, portfolio choice, uncertainty

JEL classification: C11, G11, G12

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1 Introduction

A growing body of research examines how climate change affects socio-economic outcomes, ranging from agricultural production and labor productivity to health and civil conflict.[1] Colacito, Hoffmann, and Phan (2019) show that rising temperatures can reduce U.S. economic growth by up to one-third over the next century. Climate change also affects financial markets. Hong, Li, and Xu (2019) find lower earnings growth and stock returns for food companies in countries that experience prolonged periods of drought. Bansal, Kiku, and Ochoa (2019) argue that temperature change is a source of long-run economic risk that carries a significantly positive premium in equity markets.

Investors are increasingly aware that climate change can pose significant investment risks that should be integrated in their portfolio strategies.[2] Regulators also recognize that financial institutions are exposed to environmental risk factors and encourage them to quantify these risks. For instance, new European regulations (IORP II) require pension funds to include climate risk in their own-risk assessment. However, despite their efforts, investors struggle to fully grasp the potential impact of climate change on the value of their portfolio and are searching for approaches to quantify these risks (see, e.g., the survey conducted by Krueger, Sautner, and Starks (2020)).

Existing work studies the impact of climate change on *current* asset prices. However, given the long-term nature of climate change, long-horizon investors such as pension funds are more concerned about its effects over longer holding periods. These long-term effects are difficult to measure due to three key problems: parameter uncertainty, data unavailability, and peso problems. Estimating the climate risk exposure of assets is challenging because of substantial uncertainty about the impact of climate change on asset returns, because data availability for climate risk factors is limited, and because currently available data samples may not include sufficient realizations of severe climate change effects. Robert Stambaugh summarizes the uncertainty about the long-horizon impact of climate change on asset prices by noting that when we "expand the horizon to the next several

[1] Dell, Jones, and Olken (2014) provide an overview of work on the economic impact of climate change. A more recent overview of the climate finance literature is written by Giglio, Kelly, and Stroebel (2020).

[2] In his 2020 letter to CEOs available at `www.blackrock.com/uk/individual/larry-fink-ceo-letter`, Larry Fink notes that "climate change is almost invariably the top issue that clients around the world raise with BlackRock."

decades, the possible effects of global warming range from negligible to catastrophic."[3]

We address these issues by proposing a novel approach for measuring the impact of climate change on long-horizon equity risk and return that supplements historical data with prior information derived from economic theory. In particular, we set up a Vector Autoregression model (VAR) to analyze the long-run dynamics of equity returns and include temperature levels as a predictor in the VAR model to proxy for climate risk. Since the frequency and economic impact of past climate-induced disasters may not be representative of their future impact in a scenario of prolonged climate change, historical data may not be very informative about the impact of climate change on long-horizon returns. We therefore estimate the VAR parameters using a Bayesian approach that combines historical data with theoretical prior beliefs about return dynamics.[4]

We elicit these beliefs from the temperature long-run risk (LRR-T) model of Bansal, Kiku, and Ochoa (2019), in which rising temperatures influence asset prices by increasing the likelihood of future climate disasters that lower economic growth. Because of investors' concerns about the implications of temperature increases for future growth, climate risk can be reflected in current asset prices even though the impact of climate change in historical data is limited. By imposing a structure on the relation between temperature levels and returns, the LRR-T model provides prior information about the impact of climate change. Incorporating this information yields more precise estimates of long-term climate risks than existing methods that solely rely on past data.

Our main results are as follows. First, we show that a Bayesian investor with LRR-T beliefs perceives stock markets to be riskier over longer horizons because disasters induced by climate change reduce mean reversion in returns. Intuitively, because in the LRR-T model temperature change is persistent, climate-induced disasters tend to occur in clusters during high-temperature periods, i.e., disasters are more likely to be followed by further negative shocks to consumption and dividends than by positive shocks. As a result, the long-horizon predictive variance of returns is higher for investors who form beliefs based on the LRR-T model than for investors with an uninformative prior or with a prior based on a model that does not feature climate risk, such as the

[3]This quote is from https://www.nytimes.com/2009/03/29/your-money/stocks-and-bonds/29stra.html.

[4]As pointed out by Avramov, Cederburg, and Lucivjanska (2018), "Asset pricing theory could provide additional guidance about important aspects of the return process for which the sample evidence is not particularly informative."

original LRR model of Bansal and Yaron (2004). Second, we find that the LRR-T Bayesian investor expects higher equity risk premia. In the LRR-T model, disasters cause a persistent negative shock to risk-free rates, but the negative impact on expected market returns is transitory because equity prices rapidly adjust after a disaster strikes. Therefore, the inclusion of climate change in the model increases the risk premium.

The impact of these results is clearly visible in the optimal allocation of an investor who allocates wealth to the market portfolio and the risk-free asset. In the short-term, incorporating LRR-T prior beliefs compared to the benchmark LRR beliefs results in higher allocations to the market because of the increased expected risk premium. At longer horizons, equities become considerably riskier for the LRR-T Bayesian investor, resulting in lower allocations to the market portfolio for horizons longer than 25 quarters. These results are economically meaningful, with the long-term LRR-T Bayesian investor allocating roughly 15 percentage points more in the risk-free asset at the longest analyzed investment horizon of 100 quarters.

Finally, we show that climate change has significant impact in the cross-section. We construct portfolios that are vulnerable and non-vulnerable to climate change by sorting stocks on their exposure to historic temperatures. The LRR-T model prices climate risk, implying higher future expected returns for the vulnerable portfolio. The model based on historical data predicts the opposite. Next to this, all models forecast high correlations between short-term returns for the vulnerable and non-vulnerable portfolios, implying that diversification only becomes relevant for longer-term investments. Overall, when optimizing portfolios with vulnerable and non-vulnerable assets, we find that short-term investors solely buy the portfolio with the highest perceived risk adjusted return, which differs based on the investors' beliefs. In the long run, diversification benefits bring the LRR-T Bayesian investor to equal weights to the vulnerable and non-vulnerable portfolios.

Our work contributes to two strands of literature. First, we add to the growing body of literature that studies how climate change affects asset prices. Existing work shows that carbon emissions (Bolton and Kacperczyk (2020, 2021), Ilhan, Sautner, and Vilkov (2021), and Goergen, Andrea, Nerlinger, Riordan, Rohleder, and Wilkens (2020)), temperature levels and increases (Balvers, Du, and Zhao (2017), Barnett (2020), Addoum, Ng, and Ortiz-Bobea (2020, 2021), and Kumar, Xin, and Zhang (2019)), drought trends (Hong, Li, and Xu (2019)), and broader climate-related disasters (Correa, He, Herpfer, and Lel (2021) and Huynh and Xia (2021)) all impact asset prices. Whereas these papers focus on the short-term consequences of climate change, we study the implications for long-horizon investors. There are also more and more papers on climate risk hedging portfolios (among others, Andersson, Bolton, and Samama (2016), Engle, Giglio, Kelly, Lee, and Stroebel (2020), and Pastor, Stambaugh, and Taylor (2021a,b)). We add to these by analyzing how portfolios that are vulnerable and non-vulnerable to climate change combine in optimal portfolio choice.

Second, we extend the literature on long-horizon equity risk, a topic of which the importance for investors was recently highlighted by Hasler, Khapko, and Marfè (2019). Climate change adds to known determinants of long-run risk such as mean reversion (Barberis (2000)) and uncertainty about future expected returns (Pastor and Stambaugh (2012)). Our paper builds on the approach proposed by Avramov, Cederburg, and Lucivjanska (2018) to formulate prior beliefs about return dynamics based on economic theory. We contribute to their work by incorporating climate change as a new source of long-horizon risk and by studying its impact on optimal portfolio choice for long-term investors. We show that climate change has a significant impact on the predictive distribution of equity returns, increasing both the market risk premium and the long-horizon return variance.

The paper proceeds as follows. In Section 2 we introduce our approach for modeling long-horizon equity returns and we discuss how we incorporate prior beliefs implied by the LRR-T model in Section 3. Predictive VAR estimates, long-horizon risk and return forecasts and implications of climate change for optimal portfolio choice are presented in Sections 4 and 5 for the market portfolio, and the vulnerable and non-vulnerable portfolios, respectively. Section 6 concludes.

2 Long-horizon portfolio choice

Section 2.1 introduces the Bayesian predictive VAR model that we use to characterize the long-horizon dynamics of equity returns. Section 2.2 explains the methodology used to forecast risk and return with our VARs and the construction of optimal portfolios.

2.1 Bayesian vector autoregression

We estimate a Vector Autoregression (VAR) model on quarterly data from 1947Q1 to 2019Q4 to analyze the long-horizon dynamics of equity risk and returns, as is common in the strategic asset allocation literature (e.g., Barberis (2000) and Pastor and Stambaugh (2012)). To capture climate risk, we augment the model with a return predictor related to climate change through the inclusion of the temperature anomaly in the model. Our VAR structure, therefore, incorporates information about the impact of temperatures on asset prices. Specifically, the VAR is given as

$$\begin{bmatrix} r_{m,t+1} \\ p_{t+1} - d_{t+1} \\ r_{f,t+1} \\ T_{t+1} \end{bmatrix} = a + B \begin{bmatrix} p_t - d_t \\ r_{f,t} \\ T_t \end{bmatrix} + \epsilon_{t+1}, \qquad \epsilon_{t+1} \sim N(0, \Sigma), \tag{1}$$

where $r_{m,t+1}$ is the log market return, $p_{t+1} - d_{t+1}$ is the log price-dividend ratio of the market portfolio, $r_{f,t+1}$ is the ex ante risk-free yield to maturity and T_{t+1} is the temperature anomaly.[5] The set of VAR parameters (a, B, Σ) can be split into parts as

$$\begin{aligned} &a = \begin{bmatrix} a_r \ a_x' \end{bmatrix}', \quad a_x = \begin{bmatrix} a_{pd} \ a_{rf} \ a_T \end{bmatrix}', \\ &B = \begin{bmatrix} b_r \\ B_x \end{bmatrix}, \quad b_r = \begin{bmatrix} b_{r,pd} \ b_{r,rf} \ b_{r,T} \end{bmatrix}, \quad B_x = \begin{bmatrix} b_{pd,pd} & b_{pd,rf} & b_{pd,T} \\ b_{rf,pd} & b_{rf,rf} & b_{rf,T} \\ b_{T,pd} & b_{T,rf} & b_{T,T} \end{bmatrix}, \end{aligned} \tag{2}$$

and

$$\Sigma = \begin{bmatrix} \sigma_r^2 & \Sigma_{xr}' \\ \Sigma_{xr} & \Sigma_x \end{bmatrix}, \quad \Sigma_{xr} = \begin{bmatrix} \sigma_{rpd} \\ \sigma_{rrf} \\ \sigma_{rT} \end{bmatrix}, \quad \Sigma_x = \begin{bmatrix} \sigma_{pd}^2 & \sigma_{pdrf} & \sigma_{pdT} \\ \sigma_{rfpd} & \sigma_{rf}^2 & \sigma_{rfT}' \\ \sigma_{Tpd} & \sigma_{Trf} & \sigma_T^2 \end{bmatrix}. \tag{3}$$

[5]Since we assume that the Campbell and Shiller (1988) decomposition holds in our model, we do not include dividend growth in this VAR specification. The VAR coefficients related to dividend growth can be computed directly from the coefficient estimates for the market return and the price-dividend ratio, as is done in Avramov, Cederburg, and Lucivjanska (2018). However, in our analysis, we do not study the VAR dynamics of dividend growth.

This VAR structure is also applied to a second setting, in which we analyze the impact of temperatures on portfolio choice for portfolios that are vulnerable and non-vulnerable to climate change. For that setting, we estimate the same model, but replace the market returns r_m by the returns of these portfolios, r_v and r_{nv}.

We estimate the parameters of the VAR using a Bayesian approach that complements historical data with prior information derived from economic theory. Specifically, we impose a structure on the relation between the temperature anomaly and economic and dividend growth by specifying prior beliefs based on the LRR-T model from Bansal, Kiku, and Ochoa (2019). Our Bayesian approach to incorporating climate risk into portfolio choice has several important advantages. First, it allows for the accommodation of different investor beliefs about the pricing of climate risk in equity markets. We quantify the effect of these prior beliefs on the perceived riskiness of stock markets over different holding periods and on optimal portfolio choice for long-term investors. We consider three prior investor beliefs. First, the agnostic investor has no prior views about the impact of climate change on equity return dynamics and, therefore, lets the data speak. This is equivalent to a frequentist approach for estimating the VAR model that ignores prior information and gives full weight to the return dynamics implied by historical data.[6] Second, the climate risk believer is convinced that climate change affects equity returns in the way implied by the temperature long-run risk model. This dogmatic investor therefore assigns full weight to the model-based prior beliefs. Historical data are not taken into account by this investor. Third, the Bayesian investor has faith in her prior beliefs derived from theory, but is aware that these beliefs may be inaccurate and updates them based on observed data. She assigns equal weights to the prior and the data. Our approach easily extends to different beliefs. For example, one could analyze a climate risk denier that fixes the coefficient from market returns on the temperature anomaly in the VAR model to a value of zero.

Second, the Bayesian approach allows us to assess the impact of uncertainty about the financial impact of climate change on optimal portfolio choice. We explicitly incorporate uncertainty in portfolio optimization by integrating over the posterior distribution of parameters in the return-forecasting (VAR) model to form a predictive distribution for returns. In contrast, traditional

[6] Technically, we estimate the posterior VAR for this investor on historical data with an uninformative multivariate Jeffreys prior.

frequentist methods treat the point estimates of the model parameters as known, thereby completely ignoring estimation risk. To quantify the effect of parameter uncertainty, we compare the portfolios formed based on the Bayesian predictive distributions with and without parameter uncertainty.

We derive the implications of the LRR-T model for return dynamics by applying the framework of Avramov, Cederburg, and Lucivjanska (2018). The LRR-T model does not present an analytical solution to the VAR from Equation (1). Therefore, we simulate data from our asset pricing model and estimate the VAR on the simulated data. We simulate 250,000 samples based on the processes in Equations (7)-(10), combined with the calibration from Table 1 below. Each simulated sample matches our historical data sample of 292 quarters from 1947Q1 to 2019Q4. We compute the levels, variances and correlations of the variables in the VAR model as implied by the LRR-T model as the mean of the VAR estimations for these 250,000 samples.

Our Bayesian investors combines the parameter estimates from the VAR model estimated using historical data on stock market returns, price-dividend ratios, risk-free rates, and the temperature anomaly, with the VAR parameters implied by the simulations from the LRR-T model. For the Bayesian investor, we give the same weight to the historical data and prior information. To achieve this, we set the misspecification of the model-implied prior of the VAR parameters to match the misspecification in the historical data, by scaling the prior density to the number of observations in our sample (N). This is visible in the posterior distributions presented in Appendix A.3. The LRR-T model and its implications and calibration are discussed in more detail in Sections 3.1 to 3.5.

2.2 Long-horizon analysis

For our long-horizon analyses, we forecast returns 100 quarters ahead from the Bayesian VAR from Equation (1). When we account for parameter uncertainty, we draw coefficients from the model posterior in Appendix A.3 and use these draws for each forecast path. Without parameter uncertainty forecasts are made from the mean of 250,000 draws from the VAR posterior.

2.2.1 Exogenous temperature forecasts

To ensure that the differences in investor beliefs are based on different prior views about how temperatures impact asset prices, we use the same exogenous temperature forecast paths for each investor (agnostic, dogmatic, Bayesian).[7] Effectively, we disregard information about how the financial state variables in our VAR model impact climate change, focusing solely on how temperatures affect financials. Because of this setup, the heterogeneity in our results is not driven by the uncertainty in forecasts of future temperature levels, but by how temperatures affect financial outcomes differently in our models. We forecast out-of-sample temperatures with an auto-regressive model with one lag and no intercept, starting from the average temperature anomaly over the past five years and increasing to 2 degrees Celsius by the end of our forecast period, 100 quarters ahead. We have chosen to increase the temperature forecast to 2 degrees to match goals from, among others, the UN Paris Climate Agreement.

2.2.2 Long-horizon risk and returns

We draw posterior observations from the predictive VAR from Equation (1) from the posterior VAR distribution in Appendix A.3 with direct sampling until we have 250,000 draws for each model. Conditional on each of these draws of the VAR parameters (a, B, Σ) and the recent level of the state variables x_t we compute the variance of forecasted cumulative log returns for the market and the (non-)vulnerable portfolios, $r_{i,t,t+k}$ ($r_{i,t,t+k} = r_{i,t+1} + \cdots + r_{i,t+k}$), for horizons $k = 1, \ldots, 100$ quarters. Avramov, Cederburg, and Lucivjanska (2018) show that the overall conditional variance

[7] When we forecast temperature paths from the VARs of these investors, the mean forecasted temperatures are given in Figure 8 in Online Appendix B.2. Unlike our exogenous temperature forecasts, these results are not aligned with common beliefs about future climate change.

of the long-horizon return forecast equals

$$\begin{aligned}
&\text{Var}(r_{i,t,t+k} \mid D_t) \\
&= \underbrace{\mathbb{E}[k\sigma_i^2 \mid D_t]}_{\text{i.i.d. uncertainty}} + \underbrace{\mathbb{E}\left[\sum_{j=1}^{k-1} 2b_i(1-B_x)^{-1}(I-B_i^j)\Sigma_{xi} \mid D_t\right]}_{\text{mean reversion}} \\
&+ \underbrace{\mathbb{E}\left[\sum_{j=1}^{k-1}(b_i(1-B_x)^{-1}(I-B_x^j))\Sigma_x(b_i(1-B_x)^{-1}(I-B_x^j))' \mid D_t\right]}_{\text{uncertainty about future expected returns}} \\
&+ \underbrace{\text{Var}\left[ka_i + b_i(I-B_x)^{-1}\left((kI-(I-B_x)^{-1}(I-B_x^k))a_x + (I-B_x^k)x_t\right) \mid D_t\right]}_{\text{estimation risk}},
\end{aligned} \tag{4}$$

where $D_t = (a, B, \Sigma, x_t)$. We set $x_t = \begin{bmatrix} p_t - d_t \; r_{f,t} \; T_t \end{bmatrix}'$ to the average value of the state variables in the last five years. For the theoretical prior, there is no observed state variable to forecast from, which is why we use the population moments for the price-dividend ratio and the risk-free return. Our Bayesian investor forecasts from the average of the theoretical moments and the historical data. The temperature anomaly is always forecasted from the recently observed level in the historical data, because we use the same exogenous temperature process for all models.

There are four components in the conditional long-term return variance. First, the i.i.d. uncertainty is the constant per-period variance. Second, mean reversion causes long-horizon variance to decrease, because short-term losses (gains) are offset by longer-term gains (losses) when returns revert to their ex-ante expectation. This component is why Siegel (2008) argues that equities are less risky for long-horizon investors. The third component is uncertainty about future expected returns. The idea of this component is that predictors in the VAR model are time-varying, and uncertainty about the level of future predictors will increase our predictive variance. In our case, this includes uncertainty about the level of the future temperature anomaly. The final term, estimation risk, is the variance of the predictive expected return based on the Bayesian framework in which parameters are uncertain. We study the changes to the variance over the horizon by analyzing the variance ratio $VR_{k,i} = \frac{\text{Var}(r_{i,t,t+k}|D_t)}{k\text{Var}(r_{i,t,t+1}|D_t)}$ and its underlying components.

Next to risk, we study long-horizon returns and risk premia implied by our VAR models. For this, we forecast returns on the risky and the risk-free assets from 250,000 draws of the VAR parameters. For each draw of the VAR parameters, we make returns for horizons $k = 1, \ldots, 100$ quarters and take the mean of these return paths as expected future return.

2.2.3 Portfolio choice

We compute the optimal allocation to the risky portfolios and the risk-free asset for a buy-and-hold long-only investor. Optimal portfolios for various investment horizons k are constructed by maximizing expected power utility with respect to the predictive distribution of future stock returns.[8] Formally, the investor maximizes expected utility at time $t + k$ conditional on the estimated VAR parameters (a, B, Σ)

$$\max_{w_{t,t+k}} \quad E_t[U(W_{t+k})|(a, B, \Sigma)], \tag{5}$$

where end-of-period wealth of our buy-and-hold investor is $W_{t+k} = w_{t,t+k} R_{t,t+k}$, with $w_{t,t+k}$ the vector of optimal portfolio weights to the available assets and $R_{t,t+k}$ the cumulative return on these assets. Power utility is given as $U(W_{t+k}) = \frac{W_{t+k}^{1-A}}{1-A}$, in which $A = 5$ is the risk aversion level. We compare two investors, one who invests in the market portfolio and the risk-free asset, and one who combines portfolios that are vulnerable or non-vulnerable to temperatures with the risk-free asset. We use the numerical methodology described in Online Appendix B.1 to solve the optimal weights for investment horizon k from 1 to 100 quarters. In a nutshell, we use a grid search over possible investment weights to compute the weights that give the highest average utility over 250,000 draws of future returns forecasted from the estimated VAR parameters until horizon k. As Hoevenaars, Molenaar, Schotman, and Steenkamp (2014), we explicitly account for reinvestment risk in the risk-free asset and, therefore, allow for correlated returns between the risk-free asset and risky assets.

[8] Power utility is often assumed in related work, e.g. Pastor and Stambaugh (2012), Diris (2014), Hoevenaars, Molenaar, Schotman, and Steenkamp (2014), and Johannes, Korteweg, and Polson (2014).

3 Incorporating theoretical prior information

Section 3.1 outlines the LRR-T model that is used to form prior beliefs about the impact of climate change on financial market variables. Section 3.2 describes the data including the construction of vulnerable and non-vulnerable portfolios and Section 3.3 discusses the calibration of the LRR-T model. The implied risk premia based on this calibration are discussion in Section 3.4. Finally, Section 3.5 illustrates the implications from the LRR-T model and the simulation approach used to derive the model-implied prior beliefs for the VAR parameters.

3.1 The temperature long-run risk model

Theoretical beliefs about our VAR parameters are based on an adjusted version of the LRR-T model of Bansal, Kiku, and Ochoa (2019). The LRR-T model imposes a structure on the relation between the temperature anomaly and financial market variables such as stock market returns and price-dividend ratios. The representative investor with Epstein and Zin (1989) recursive preferences optimizes lifetime utility

$$U_t = \left[(1-\delta)C_t^{\frac{1-\gamma}{\theta}} + \delta(\mathbb{E}_t[U_{t+1}^{1-\gamma}])^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\gamma}}, \tag{6}$$

with C_t aggregate consumption at time t, $\delta \in (0,1)$ the investor time preference, γ the coefficient of risk aversion, $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$, and ψ the elasticity of intertemporal substitution (IES). The log of aggregate consumption growth ($\Delta c_{t+1} = \log(C_{t+1}/C_t)$) and log dividend growth of the portfolio i[9] ($\Delta d_{t+1,i} = \log(D_{t+1,i}/D_{t,i})$) follow

$$\begin{aligned}
\Delta c_{t+1} &= \mu_c + \sigma_t \eta_{t+1} + X_{t+1}, \\
\Delta d_{t+1,i} &= \mu_d + \pi_d \sigma_t \eta_{t+1} + \phi_i X_{t+1} + \varphi_d \sigma_t u_{t+1}, \\
\sigma_{t+1}^2 &= \bar{\sigma}^2 + \nu(\sigma_t^2 - \bar{\sigma}^2) + \sigma_w w_{t+1}, \\
&\eta_{t+1}, u_{t+1}, w_{t+1} \sim N i.i.d.(0,1),
\end{aligned} \tag{7}$$

[9]Portfolio i is the market or the portfolio (non-)vulnerable to temperature innovations, $i = m, v, nv$.

with mutually independent shocks η_{t+1}, u_{t+1}, and w_{t+1} scaled by time-varying volatility σ_t. X_{t+1} is the adverse economic impact of temperature-driven disasters on consumption and dividend growth, based on a disaster process N_t with Poisson distributed increments as

$$\begin{aligned} X_{t+1} &= \rho X_t + d\Delta N_{t+1}, \\ \Delta N_{t+1} &\sim \text{Poisson}(\lambda_t = \Delta t(\lambda_0 + \lambda_1 T_t)), \end{aligned} \tag{8}$$

where $\rho < 1$ is the persistence of economic disaster impact, $d < 0$ is the initial disaster-related growth shock to consumption and dividends and T_t is the temperature level, in turn based on the atmospheric carbon concentration ε_t as

$$\begin{aligned} T_{t+1} &= \chi\varepsilon_{t+1}, \\ \varepsilon_{t+1} &= \nu_\varepsilon \varepsilon_t + \mu_\varepsilon + \Theta(\mu_c + \sigma_t \eta_{t+1}) + \sigma_\zeta \zeta_{t+1}, \\ \zeta_{t+1} &\sim Ni.i.d.(0, 1). \end{aligned} \tag{9}$$

This structure allows for a feedback loop between consumption growth and the atmospheric carbon concentration, by including μ_c and η_{t+1} in the process for the latter. We assume that the log of the wealth-consumption ratio z_t and the log of the price-dividend ratio of portfolio i, $z_{t,i}$, are given as

$$z_{t(,i)} = A_{0(,i)} + A_{1(,i)} T_t + A_{2(,i)} X_t + A_{3(,i)} \sigma_t^2. \tag{10}$$

Portfolios differ in the exposure of future dividend growth on disaster impact X_{t+1}, through the portfolio-specific parameter ϕ_i in Equation (7), which is higher for portfolios more vulnerable to temperatures. The analytical solution of the price-dividend and wealth-consumption ratios, using the Campbell and Shiller (1988) decomposition, is presented in Appendix A.1. We discuss the adjustments made to the LRR-T model of Bansal, Kiku, and Ochoa (2019) in Appendix A.2.

Next to the LRR-T model, we also analyze the impact of the normal long-run risk (LRR) structure on equity risk, return, and portfolio choice. For this, we use the LRR specification from Bansal, Kiku, and Yaron (2012), calibrated to match our sample. Our Bayesian investor that includes LRR prior beliefs forms the benchmark for our LRR-T Bayesian investor, highlighting the impact of climate change on long-horizon outcomes.

3.2 Data

Market returns, dividend growth, and price-dividend ratios are from the Irrational Exuberance dataset available on Robert Shiller's website.[10] As a proxy for market returns we use the monthly real log returns including dividends on the S&P 500 index. Dividend growth is the log difference of the monthly real dividends on the market portfolio. The log price-dividend ratio is the log difference between the real S&P 500 price and the corresponding monthly real dividend.

Monthly ex ante real risk-free returns are constructed following Beeler and Campbell (2012). We use the seasonally unadjusted consumer price index (CPI) from the Bureau of Labor Statistics to construct quarterly and yearly inflation as the log difference between the CPI levels at the end of the current period and the end of the previous period. To construct ex post real risk-free yields we subtract the quarterly log inflation from the log CRSP Treasuries three month risk-free yields. Ex ante risk-free rates are the predicted value from the regression of ex post real risk-free yields on an intercept, nominal risk-free yields and the annual log inflation divided by four.

We obtain average monthly land-based U.S. temperature anomalies from the nClimDiv dataset of the National Oceanic and Atmospheric Administration (NOAA).[11] We transform these anomalies to degrees Celsius, as in Bansal, Kiku, and Ochoa (2019).

Consumption data is from National Income and Product Accounts (NIPA). Quarterly per capita real consumption growth is the seasonally-adjusted aggregate nominal consumption expenditures on nondurables and services (NIPA Table 2.3.5), adjusted with the price deflator series from NIPA Table 2.3.4 and divided by population (from NIPA Table 2.1).

We construct portfolios that are vulnerable or non-vulnerable to temperatures. For this, we use log real monthly returns of all stocks in CRSP excluding penny stocks (defined as stocks with price below \$5 at quarter end). The construction of these portfolios is discussed in Section 5.

[10] http://www.econ.yale.edu/~shiller/data.htm.

[11] Temperature anomalies are the monthly average temperatures minus the average temperature in that same month for our base period 1901-1946. This base period is chosen to end just before our sample, to avoid look-ahead bias.

The sample period is 1947-2019.[12] All variables are monthly and time-aggregated to quarterly using the methods from Bansal, Kiku, and Yaron (2016), except for consumption growth. Consumption growth is quarterly and available from 1947Q2, because monthly consumption data is unavailable in the early years of our sample.

3.3 Model calibrations

We calibrate the LRR-T model with the calibration parameters given in Table 1. In our calibration, the temperature anomaly is zero in expectation at the start of our simulated sample, with a long-term temperature expectation that is one degree Celsius higher. This scenario is chosen because it closely matches our historical data, where we observe an increase in the temperature anomaly roughly from zero to one degree Celsius.[13] Next to temperatures, we calibrate the LRR-T model to match economic growth and financial moments for our sample 1947Q1-2019Q4. Second, we allow for portfolio-specific impact of climate change by adjusting the dividend growth loading on climate disasters ϕ_i to increase or decrease vulnerability towards climate change. Specifically, we include the market portfolio in the baseline calibration and use ϕ_v and ϕ_{nv} for the vulnerable and non-vulnerable portfolios, respectively. Our benchmark is based on a second informative prior, from the LRR model of Bansal, Kiku, and Yaron (2012) that does not feature climate risk.[14]

[Table 1 about here.]

[Table 2 about here.]

The first and second moments of observed economic growth, price-dividend ratios and returns in the data are compared with the implications from the LRR(-T) model calibrations in Table 2. The moments from the LRR(-T) models are the average moments of 250,000 simulations of 936

[12]Our starting date follows Barnett (2020) and balances the need for a longer sample to make more accurate long-term return forecasts with the fact that data from periods preceding the general awareness of climate change is likely uninformative about the impact of climate change on long-horizon equity risk.

[13]In our sample, the temperature anomaly is 0.1 in 1948 and 1.2 in 2019. Alternative sources of temperature anomalies show more smoothed versions that are closer to the 0-1 we use in our calibration.

[14]We use a different sample and re-calibrate several parameters in the Bansal, Kiku, and Yaron (2012) model to match the observed data moments of our sample period (1947-2019) better. Specifically, our calibration deviates from the original by setting $\rho = 0.981$, $\mu_d = 0.0021$, $\phi = 2.0$, $\pi = 2.0$, and $\varphi = 5.0$. Avramov, Cederburg, and Lucivjanska (2018) also elicit prior beliefs from this version of the LRR model.

months from the model, time-aggregated to match the 73 years of our sample after a burn-in period of 60 months. The LRR-T model fits the data well in most aspects, similar to the original LRR model. There are two main moments the model does not match the data. First, as is commonly observed with LRR models, the price-dividend ratio level and variance is too low. For example, Bansal, Kiku, and Yaron (2012) report expected price-dividend ratios 9% below the data estimate in their LRR population moments, comparable to our results. Second, the expected temperature anomaly is higher in the LRR-T simulations than observed in our sample. This is driven by the concave increase in the LRR-T autoregressive structure for the temperature level, while historical temperature increases are more linear. Therefore, the temperature level in the early years of our LRR-T simulations is higher than in the corresponding decades in the data.

3.4 Model implied risk premia

With the calibrations for the LRR(-T) models, the theoretical market risk premium for the LRR and LRR-T models can be decomposed. The market risk premium in the LRR-T model equals

$$\begin{aligned} \ln \mathbb{E}_t[R_{m,t+1}] - r_{f,t} &= -\mathrm{Cov}_t(m_{t+1} - \mathbb{E}_t[m_{t+1}], r_{m,t+1} - \mathbb{E}_t[r_{m,t+1}]) \\ &= \lambda_\eta \beta_{m,\eta} \sigma_t^2 + \lambda_\zeta \beta_{m,\zeta} \sigma_\zeta^2 + \lambda_w \beta_{m,w} \sigma_w^2 + \lambda_X \beta_{m,X} (\lambda_0 \Delta t + \lambda_1 \Delta t T_t), \end{aligned} \tag{11}$$

with the derivation and analytical solutions for lambdas and betas given in Appendix A.1.

There are four components of the market risk premium. The first term, related to shocks η_t, is the growth premium, as it is directly related to shock to consumption growth. The second component is the temperature premium, related to the uncertainty in the temperature process that impacts the future expected disaster-occurrence. Note that this term is related to the uncertainty in the temperature process, not to the actual temperature level. Therefore, even when the temperature anomaly is forecasted to decrease, this premium will remain stable as a reward for the risk of future changes in the number of expected disasters. Third, we have a volatility risk premium that affects the time-varying volatility of shocks to consumption growth. Finally, the last component is the disaster premium, which is based on the variance of the current disaster process, that is in turn a function of the temperature anomaly. Because of this final component, higher temperature levels increase the market risk premium directly. The growth, temperature, and disaster premia are introduced in Bansal, Kiku, and Ochoa (2019). The volatility risk premium is common in LRR models when time-varying standard deviation is included, but not a component in the original LRR-T model.

Comparing the LRR and LRR-T risk premia, we see a similar structure, but the focus clearly changes to a climate related risk premium in the latter model. In the LRR model from Bansal, Kiku, and Yaron (2012), there are three components of the risk premium: short-run risk, long-term growth risk, and volatility risk. The first component has the same structure in the LRR and LRR-T models, and corresponds to the growth premium discussed above. The long-term growth risk is related to the exposure to shocks in the persistent component of the LRR model, and as such comparable with the disaster premium in the LRR-T model. Finally, volatility risk impacts the risk premium in two ways in the normal LRR model: by introducing (short-term) noise in the direct shock to consumption growth η, and by affecting the size of the shock to the persistent component. The first aspects of the volatility risk corresponds to the same component of the LRR-T risk premium. The second aspect is comparable to the LRR-T temperature premium, which is based on the noise in the temperature anomaly, a measure for the future expected disaster shock impact.

Based on our calibrations, we find that our LRR model implies that 17% of our risk premium is driven by short-term risk, 32% by long-term growth risk, and 51% by long-run volatility risk. In the LRR-T model, 8% of the risk premium is driven by the growth premium, 63% by the temperature premium and 29% by the disaster premium. The volatility risk premium has negligible impact on the risk premium, since it only adds noise to the short-term consumption growth shocks. However, it is useful to have time-varying volatility in the model to better match the second moment of the market return in our LRR-T calibration. There are two main takeaways from this decomposition. First, the LRR-T risk premia are driven by components that are, in their structure, similar and comparable to what the LRR model implies. Second, temperature levels and climate-related disasters are a very important part of the financial implications of the LRR-T model.

3.5 Model simulations

To illustrate how temperatures impact financial performance in the LRR-T model we show a single simulation from our calibrated model in Figure 1. In this simulation we see that temperatures increase over the sample (in line with the calibrated trend), with quite some variance in the temperature process as is observed in the historical data. In expectation, temperature anomalies start at 0 and increase to 1, but we observe both much higher and lower temperature anomalies in the simulation, with a coincidental low temperature anomaly at the end of our simulated sample. With increases in temperatures, the expected occurrence of future disasters that affect future consumption and dividend growth rates is increased. Price-dividend ratios and risk-free rates respond to these expected future adverse events with immediately decreases that are mostly visible in the price-dividend ratio. Climate disasters start occurring after roughly 250 months in this simulation, most clearly visible in the risk-free rate.[15] These disasters have an immediate and persistent impact on risk-free rates and price-dividend ratios. For risk-free rates, persistent impact is caused by the slow response of the wealth-consumption ratio to disasters. The stability of the wealth-consumption ratio, especially in a long-run risk setting, is also documented by Lustig, VanNieuwerburgh, and Verdelhan (2013). In market returns, we do not observe persistent impact of disasters, as decreasing prices keep future market returns relatively stable. There is, however, significant transitory impact from climate disasters on market returns, as the most negative market returns observed in our simulation *are* caused by disasters. Overall, the market risk premium increases after disaster occurrence, because risk-free returns are persistently lower while the market rebounds quickly after the initial transitory shock.

[Figure 1 about here.]

The population moments from 250,000 simulations of the baseline LRR-T model imply the VAR structure shown in Table 6 in Online Appendix B.2. In this estimation, all coefficients are significant by construction, since we base the VAR in this table on the population moments

[15] We note that, in this simulation, we observe disasters relatively late by coincidence - in expectation we would observe roughly one disaster each every 100 months. Overall, there are 10 disasters in this simulation, which is what we would expect.

from the model without allowing for misspecification. First, relations between market returns and financial state variables are as expected. Increases in price-dividend ratios decrease expected market returns and increases in risk-free rates increase expected market returns. This VAR structure differs from previous literature by the inclusion of the temperature anomaly as a predictive variable. Temperature levels have a strong negative impact on next period expected market returns. The coefficient from market returns on the temperature anomaly is economically large: a temperature anomaly of 1 degree Celsius implies that quarterly expected market returns are decreased by 0.61%, or 2.4% annually. A one standard deviation shock to the temperature anomaly decreases quarterly expected market returns by 0.51%, or 2% annually. This initial impact on market returns is partly offset in long-horizon forecasts, because the temperature increase also decreases price-dividend ratios, but that effect is smaller. The temperature anomaly is highly persistent, which is implied by the LRR-T model because we calibrate a positive trend in the temperature levels. Overall, this VAR shows that the LRR-T model indeed imposes a structure on the impact of temperature levels on equity risk and return.

4 Empirical results: market portfolio

Thus far we have examined the effect of climate change on financial markets through the lens of the LRR-T model. We now discuss how these theoretical implications affect the empirical estimates of a Bayesian investor that combines the market portfolio with a risk-free asset. Section 4.1 presents predictive regression estimates for our investors with different prior beliefs based on the LRR-T model, as well as a benchmark investor who forms prior beliefs from the LRR model without climate change. Sections 4.2 and 4.3 discuss the implications of these estimates for long-horizon risk and return of these VAR models. Section 4.4 combines these results in the optimal portfolio choice of our investors.

4.1 Predictive regressions

We estimate the quarterly predictive VAR from Equation (1) on the sample from 1947Q1 to 2019Q4 for four investor types. The first three investor types are the agnostic (data-based) investor, the dogmatic (climate risk believer) investor, and the Bayesian LRR-T investor discussed in Section 2.1. Finally, we analyze a benchmark Bayesian LRR investor that forms prior beliefs based on the LRR model without climate change. Table 3 shows the posterior mean of 250,000 draws of the VAR coefficients for these investor types, along with their posterior t-statistics.

In panel A of Table 3, we show the posterior VAR for the agnostic investor, based on historical data with the temperature anomaly as a state variable. From the coefficient estimates we observe that price-dividend ratios have negative forecasting power on market returns, while increases in risk-free yields imply higher market returns, as expected. We observe statistically insignificant coefficient estimates on the temperature anomaly. This could be a result of large uncertainty about the impact of climate change on financial markets, because of parameter uncertainty or unavailability of sufficient disasters in historical data. This uncertainty is one of the key reasons to take prior beliefs into account in a Bayesian setting, which helps estimation precision. On the other hand, the coefficient estimates on the temperature anomaly are economically very meaningful. A one standard deviation shock to the temperature anomaly implies a 0.74% change in the forecasted quarterly market return, or almost 3% annually. Based on forecasts for future temperature anomalies of up to two degrees Celsius, impact of temperatures on market returns would be even larger, especially in long-horizon forecasts. Other than market returns, financials are not strongly affected by temperatures in the data. The temperature process itself is included for completeness but not used for forecasts, where an exogenous temperature process is used, as discussed in Section 2.2.1.

The opposite perspective is a dogmatic investor that bases her beliefs about the VAR model completely on the LRR-T prior. Panel B of Table 3 reports the posterior VAR for this climate risk believer. The VAR coefficients are almost identical to the population estimates for the VAR parameters reported in Table 6. As before, we observe a negative loading from market returns on the temperature anomaly. In the LRR-T model, increasing temperatures decrease the price-dividend ratio, increasing market returns. However, this effect is offset by the realisation of large negative returns around climate disasters, which occur more often with high temperature levels. In our simulations, the second effect is stronger. The difference with the population estimates are in the decreased statistical significance of the results. We introduce model misspecification in these estimates to match the information in the prior beliefs to the historical data sample from 1947Q1 to 2019Q4, as described in Section 2.1. Because of the noise in the agnostic VAR based on historical data, this results in large uncertainty in the VAR of the dogmatic investor as well. However, we still observe economically meaningful negative impact from increased temperatures on financial outcomes.

Comparing panels A and B, it is clear that historical data and economic theory have opposing implications about the impact of climate change on financial performance, which makes it useful to include both views in a Bayesian analysis. A key difference between the agnostic and dogmatic investors is that the former does not show a relation between both the price-dividend ratio and risk-free returns and the temperature anomaly, while this relation is reasonably strong for the latter. This is driven by the impact of climate disasters on risk-free returns in the LRR-T model. In that model, disasters result in additional savings, decreasing risk-free returns.

Panel C of Table 3 reports the posterior VAR for the Bayesian investor that gives equal weights to implications from historical data (panel A) and the baseline LRR-T prior (panel B). As expected, the posterior coefficients and correlations of this VAR are generally in between the reported values in panel A and panel B. Since panels A and B present opposite results, the Bayesian investor has a relatively small loading on the temperature anomaly, which is visible in the small absolute posterior coefficients. This small coefficient *does* have significant impact on predictive distributions of market returns, especially for long-horizons. Small changes in the predictive VAR parameters become increasingly important after several quarterly forecasts. Next to this, combining historical data with LRR-T simulated data increases the precision of the parameter estimates, which decreases the long-horizon variance of return forecasts.

Finally, panel D of Table 3 is the benchmark Bayesian LRR investor, that combines implications from historical data with the Bansal, Kiku, and Yaron (2012) LRR prior. As expected, the general structure of the VAR is very similar to Panel C, the only real difference is the fact that the temperature anomaly is not included in the model. Therefore, comparing the forecasts from this investor with the implications from the Bayesian LRR-T investor really highlights the impact of the temperature anomaly on our models, instead of other differences in the LRR and LRR-T models.

[Table 3 about here.]

4.2 Long-horizon risk

We now use the 250,000 draws of the VAR coefficients around their posterior mean in Table 3 to forecast long-horizon variance of market returns. For this, we combine draws of the coefficients with the exogenous temperature process discussed in Section 2.2.1 in the long-horizon variance formula from Equation (4) to forecast variance ratios for horizons up to 100 quarters. In Figure 2, the per period variance by horizon is given for the agnostic (data-based), dogmatic (LRR-T based), and Bayesian LRR and LRR-T investors (combining data with model based priors).

Within a few quarters, the variance ratios quickly diverge. In the data, we find strong mean reversion, which is also documented by, among others, Barberis (2000) and Siegel (2008). Intuitively, current low (high) returns are offset by future high (low) returns, because of predictability in returns in the data combined with negative correlation between current and future returns. Therefore, variance ratios quickly decrease when the horizon gets longer. As opposed to earlier work, the large coefficient from market returns on the temperature anomaly increases the uncertainty about future expected returns for the agnostic investor, as the future temperature level is highly uncertain. At the longest horizon of our forecasts, the decrease in the additional mean reversion combined with the continuously increasing uncertainty about future expected returns increases the variance ratio again, as compared to horizons between 50 and 75 quarters. Overall, the variance ratios over the horizon of our agnostic investor is similar to the results in Avramov, Cederburg, and Lucivjanska (2018), but increased predictability in our setting from including the temperature anomaly in our model increases both mean reversion and uncertainty about future expected returns.

At first sight on the overall variance ratios, the benchmark LRR Bayesian investor seems to be close to the data-based agnostic investor. However, the underlying components driving long-horizon variance for these two investors are very different. The key difference is that the data implies stronger predictability from the price-dividend ratio, which increases both mean reversion and uncertainty about future expected return compared to the Bayesian LRR investor.

The dogmatic investor with its LRR-T prior shows very high long-horizon variance ratios, with per period market volatility predicted up to 2.75 times higher than the one-quarter ahead forecast. There are two key drivers of the difference with the agnostic investor. First, mean reversion is weaker. Intuitively, mean reversion decreases because current low returns caused by climate-induced disasters are followed by future low returns caused by even more disasters. This effect is driven by the fact that disasters in the LRR-T simulations occur in clusters, in periods with high temperature levels. Therefore, the negative correlation between current and future returns that is needed for mean reversion is much weaker than in the data. Second, estimation risk has increased significantly, because the disasters in the LRR-T simulations increase the uncertainty around the parameter estimates.

Influenced by the LRR-T prior, the LRR-T Bayesian investor has relatively high variance ratios, monotonically increasing with the investment horizon. At the longest horizon of 100 quarters, the variance ratio of the LRR-T Bayesian investor is twice as large as that of the benchmark LRR Bayesian and the agnostic investors. The underlying components of the variance ratio are between what is implied by the data and the prior beliefs, except for the uncertainty about future expected returns. Lower uncertainty is driven by the relatively small coefficient from market returns on the temperature anomaly for the Bayesian investor, a logical result from the opposite signs of corresponding coefficient for the agnostic and dogmatic investors. Even though the coefficient on the temperature anomaly is smaller for the LRR-T Bayesian investor, the inclusion of temperatures in the model is sufficient to increase uncertainty about future expected returns compared to the benchmark LRR investor.

[Figure 2 about here.]

For our asset allocation decision, the riskiness of the short-term bond returns is also relevant to consider. We assume that our short-term bond is risk-free in the sense that there is no credit risk. However, over longer horizons, reinvestment risk affects investments that are risk-free in the short run. The variance of the risk-free return relative to the risk of the market return is presented in Figure 9 in Online Appendix B.2. For all models, risk-free returns have more variance with longer horizons, as reinvestment risk increases with the horizon. Both the data and the LRR-T based models imply similar relative riskiness between risk-free and market returns. The Bayesian LRR investor forecasts less reinvestment risk for the risk-free asset. This observation aligns with the calibration moments in Table 2, where LRR based risk-free return simulations are less volatile than in the data and the LRR-T model.

Overall, including climate change through LRR-T prior beliefs results in higher predictive long horizon risk for the LRR-T Bayesian investor investing the market portfolio. This effect is large, with long-run variances up to twice as high as as the benchmark LRR Bayesian investor. Next to market returns, the LRR-T Bayesian investor also predicts more reinvestment risk in the risk-free asset.

4.3 Long-horizon returns

Predictive returns also differ significantly for the four different investors and by horizon. Figure 3 shows the average forecast of risk-free returns, market returns and market risk premium from 250,000 simulated return paths. Each return path is forecasted from a draw of the VAR posterior distribution for each investor type to account for parameter uncertainty, combined with the exogenous temperature process discussed in Section 2.2.1.

The risk-free return forecasts are initially all very similar and close to zero, in line with the current observed short-term T-bill rate. With forecasts over the horizon, we see opposite trends for the agnostic and dogmatic investors. Forecasts from the data push long-term risk-free returns towards the historical average, which is higher than the current risk-free rate levels. In the LRR-T model, however, future temperature increases decrease the risk-free rate. This reflects the negative impact that climate disasters have on the risk-free returns in the LRR-T simulations. Disasters persistently affect future consumption and dividend growth and agents save to offset this, increasing the demand for the risk-free asset, decreasing its return. Since the risk-free rate is very persistent and temperatures keep increasing in our forecasts, the dogmatic investor predicts decreasing risk-free returns over our horizon. We note that this decrease is not set to continue indefinitely if one extends the horizon, since risk-free rates are not 100% persistent and climate models generally do not forecast unbounded temperature increases. The LRR-T Bayesian investor forecasts risk-free returns in between the data and the LRR-T model. Without temperature increases in the model, the benchmark LRR Bayesian investor forecasts risk-free returns similar to the agnostic investor.

The predictive market returns reported in Figure 3 are very similar for the agnostic and LRR Bayesian investors, both increasing by horizon. The agnostic investor has positive VAR coefficients from market returns on the temperature anomaly and the risk-free returns, as visible in panel A of Table 3. The increase in predictive market returns over the horizon is, therefore, driven by the increase in predictive risk-free returns and the increasing temperature levels in the exogenous temperature process. For the LRR Bayesian investor, temperatures are not included in the model. However, a similar horizon pattern for predictive market returns is driven by a larger loading on the increasing risk-free returns, as visible in panel D of the same table.

The most surprising of the return forecasts is the quick increase in the short-term market returns expected by the dogmatic investor. This is a direct result of the implementation of an exogenous temperature process, where the starting value of our out-of-sample temperature anomaly is the recently observed level in the data. This temperature level is higher than in the LRR-T simulated data used by the dogmatic investor, which is why there is a jump in the temperature levels from the in-sample estimation to the out-of-sample forecasts. This initial jump in our forecasts quickly decreases the price-dividend ratio, resulting in a sharp increase in expected market returns. We do not feel that this is a realistic forecast and, therefore, do not conclude anything from the first ten quarters of the LRR-T prior forecasts.[16] At longer horizons, increasing temperatures combined with decreasing risk-free returns drive expected market returns down. This corresponds well to the observed negative disaster-related returns in the LRR-T simulations, as presented in Figure 1 above.

The market returns predicted by the LRR-T Bayesian investor are stable over the horizon, as increased temperatures balance the negative impact from decreases in the risk-free returns. In other words, the direct impact from the forecasted temperature increase on market returns is offset by its indirect impact on market return through the risk-free asset.

All four models imply that the market risk premium increases with the investment horizon,

[16]One could argue that this result implies that the exogenous temperature process should not be the same for all our models. However, we believe that it is more important to keep future temperature processes similar to compare the differences in the implications of climate change between the models than to make these short-term market return forecasts more stable.

although the underlying mechanism differs. The agnostic investor forecasts increasing market returns with the horizon, mainly driven by decreasing price-dividend ratios towards the historical average and increasing temperatures. The LRR Bayesian benchmark investor shows similar risk premium forecasts, at a lower level because of slightly higher risk-free returns. For the dogmatic and LRR-T Bayesian investors, decreasing risk-free rates are an important aspect of the increasing risk premium over the horizon. While the horizon effects for the market risk premium are similar for all models, the risk premia are at different levels.

[Figure 3 about here.]

Finally, we discuss the correlations of the returns on the risk-free asset and the market portfolio. Figure 10 in Online Appendix B.2 documents that, for all our investors, these returns start relatively uncorrelated and increase over the horizon. This increased correlation is mainly driven by the positive coefficient from market returns on lagged risk-free returns in the VAR models of Table 3.

Overall, the LRR-T Bayesian investor predicts higher market risk premia over all investment horizons than the agnostic and LRR Bayesian investors. This effect is driven by predictive risk-free returns that decrease over the horizon, combined with relatively high short-term market return predictions.

4.4 Portfolio choice

We have now documented two key results. First, the LRR-T Bayesian investor predicts significantly higher long-horizon riskiness of the market portfolio than our benchmark LRR Bayesian investor, because of the uncertainty in the temperature anomaly forecast and increased estimation risk. Second, including beliefs from the LRR-T model results in higher predictive market risk premia, because forecasted climate-related decreases in consumption growth negatively affect long-horizon risk-free returns. Next to these key results, the LRR-T model implies relatively more reinvestment risk and higher correlations between market and risk-free returns than in the data. These results each have implications for portfolio choice, as we discuss in this section.

We analyze a long-only investor that can invest in the market portfolio and a risk-free asset with reinvestment risk. The top panel of Figure 4 shows the optimal allocation to equities for the agnostic, dogmatic, and Bayesian investors. The optimal weight in equities increases quickly with the investment horizon for the agnostic investor, because there is very strong mean reversion in the related predictive VAR. At the longest horizon market returns become riskier again and predictive risk-free returns become higher, which brings the optimal weight to the market portfolios down. The dogmatic investor initially sees a quick increase in the optimal allocation to the market portfolio driven by the sharp increase in the market risk premium, a consequence of our exogenous temperature process. Over the horizon, the dogmatic investor keeps allocating less and less to the market because the perceived long-horizon risk of the market portfolio increases quickly.

As expected, the allocation to the market portfolio of the LRR-T Bayesian investor is in between the allocations of the agnostic and dogmatic investors. Comparing the LRR-T and LRR Bayesian investors, we find that the inclusion of the temperature anomaly in the model has economically meaningful impact on optimal portfolios, which changes over the horizon. The investor with LRR-T prior beliefs has higher weights to equities than the LRR Bayesian investor for horizons of up to roughly 25 quarters, with the opposite result for longer investments horizons. This is mainly driven by the two key results discussed above: the inclusion of climate change in the model implies higher market risk premia for all horizons, but risk increases considerably over the horizon. Therefore, the allocation is initially higher, but becomes lower when the increased risk outweighs the higher market risk premium. Especially in the long run, these differences are economically meaningful: the optimal weights to equities are roughly 20% lower for the LRR-T Bayesian investor that invests for 100 quarters, compared to the benchmark LRR Bayesian investor.

The riskiness of the market portfolio and the risk-free asset are affected very differently by parameter uncertainty. In Hoevenaars, Molenaar, Schotman, and Steenkamp (2014), parameter uncertainty does not have significant impact on portfolio choice, but with the inclusion of the temperature anomaly in our model we find higher estimation risk for equities. Reinvestment risk of risk-free returns is, on the other hand, mainly driven by uncertainty about future expected returns, because of the loadings from risk-free returns on the temperature anomaly. Therefore, when parameter uncertainty is accounted for, equity returns become considerably more risky, relative to risk-free returns. The longer the investment horizon, the more important the inclusion of parameter uncertainty becomes for our portfolio choice. In the bottom panel of Figure 4, we therefore observe similar weights to the market portfolio for short horizons with and without parameter uncertainty, but significantly lower allocations to equities in the long run. The differences between the allocations of the different investors are similar to before, with the same key results driving the allocation over the horizon.

[Figure 4 about here.]

5 Empirical results: (non-)vulnerable portfolios

We have now analyzed the implications of taking the temperature anomaly into account for investors that combine the risk-free asset with the market portfolio. In this section, we extend our setting to allow for cross-sectional differences in long-horizon portfolio choice. Specifically, we now analyze investors that include portfolios that are either vulnerable or non-vulnerable to temperatures in their investment universe, instead of the market portfolio.

We construct portfolios that are vulnerable or non-vulnerable to temperatures based on a sort of contemporaneous exposure from stock returns on the temperature anomaly, building on the work of Barnett (2020). We run a five-year rolling window regression (with at least 2.5 years of available data) of the log real monthly returns of all stocks in CRSP excluding penny stocks (defined as stocks with price below $5 at quarter end) on the temperature anomaly and log real market returns. We then sort the stocks on their exposure to the temperature anomaly and take the top (bottom) quintiles as the (non-)vulnerable portfolios. We take value weighted returns of the stocks within these portfolios in the quarter after our regression as the quarterly return of our vulnerable and non-vulnerable portfolios.

Table 4 reports the summary statistics of these portfolios. Throughout our sample, the non-vulnerable portfolio has given slightly higher returns, but at the cost of higher standard deviation and kurtosis. The historical risk adjusted returns are very similar.

[Table 4 about here.]

The rest of this section is structured as follows. First, we discuss the predictive regressions for the vulnerable and non-vulnerable portfolio returns in Section 5.1. The implications from these models on long-horizon risk, return and portfolio choice is discussed in Sections 5.2 and 5.3.

5.1 Predictive regressions

Table 5 shows the predictive regressions for the vulnerable and non-vulnerable portfolio returns, the state variables in the model have the same structure as in Table 3. The agnostic investor in panel A has similar predictive regressions for the vulnerable and non-vulnerable portfolios, with a small difference in the temperature coefficient between the two. As constructed, non-vulnerable portfolio returns increase have a higher loading on the temperature anomaly than vulnerable portfolio returns. The difference between the two initially seems small, but with temperature anomalies between 1 and 2 degrees Celsius this results in 0.16% to 0.32% higher quarterly returns for the non-vulnerable portfolio purely based on the temperature anomaly.

Panel B of Table 5 reports the predictive regression for the dogmatic investor. As for the market portfolio, both equity portfolios have negative loadings on the temperature anomaly, with more impact on the vulnerable than on the non-vulnerable portfolio. In this setting, however, there are also considerable differences in the loadings on the intercept, price-dividend ratio and the risk-free rate. The higher loading from the vulnerable portfolio on these components is driven by the fact that climate-disasters hit the vulnerable portfolio harder, which drives additional correlation with both the risk-free rate and the price-dividend ratio.

Combining historical data and LRR-T prior beliefs results in the LRR-T Bayesian regression in panel C of Table 5. In this setting, the temperature anomaly has less impact on equity returns, because of the opposite signs on the prior and the data. Surprisingly, the Bayesian investor has a higher coefficient from the vulnerable returns on the temperature anomaly than from the non-vulnerable portfolio returns, which is not the case in the prior and the data. The difference in the loadings on the temperature anomaly remains economically meaningful.

[Table 5 about here.]

5.2 Long-horizon risk and return

The predictive regressions from Table 5 forecast the variance ratios for the (non-)vulnerable portfolios as presented in the top panels of Figure 5. Comparing these results to the predictive variances of the market portfolio from Figure 2, we observe several differences. Most importantly, the agnostic investor predicts higher long-horizon variance ratios for our sorted portfolios than for the market portfolio. We find that these high variance ratios are mainly driven by slightly decreased predictability, which decreases mean reversion and to a lesser extend uncertainty of future expected return. Second, the estimation risk of the agnostic investor has increased significantly.[17] The dogmatic and LRR-T Bayesian investors predict similar variance ratios for the market portfolio and our sorted portfolios. The vulnerable portfolio shows slightly more mean reversion than the non-vulnerable portfolio, decreasing the overall variance ratio. In addition to these variance ratios that document variances of the sorted portfolios over the horizon, Figure 13 in Online Appendix B.2 documents how the vulnerable and non-vulnerable portfolios compare in their relative variance. We find that the agnostic and LRR-T Bayesian investor initially perceive the non-vulnerable portfolio as riskier, with the opposite result for the dogmatic investor.

The forecasted risk premia for the (non-) vulnerable portfolios are given in the bottom panels of Figure 5. The dogmatic investor predicts risk premia for the non-vulnerable portfolio that are roughly 0.30% per quarter higher than that of the vulnerable portfolio, which aligns well with the difference in the temperature anomaly coefficients in panel A of Table 5. The dogmatic investor predicts significantly higher risk premia for the vulnerable portfolio. In the LRR-T model, persistent temperature risk is priced, which implies that in-sample, the vulnerable portfolio also had significantly higher risk premia. It is not unexpected that this is also the case in the out-of-sample forecasts. Finally, the LRR-T Bayesian investor also forecasts higher returns for the vulnerable portfolio, but these are considerably closer to those of the non-vulnerable portfolio.

[Figure 5 about here.]

Next to risk and return, we consider the predictive correlations between the vulnerable and

[17] The decomposition of the variance ratios and its underlying components is found in Figures 11 and 12 in Online Appendix B.2 for the vulnerable and non-vulnerable portfolios, respectively.

non-vulnerable portfolios, as well as the risk-free asset. These are presented in Figure 6. We find that over the horizon, the vulnerable and non-vulnerable portfolio returns are less correlated with eachother, especially for the agnostic investor. Therefore, with longer investment horizons, there are more diversification benefits between the two risky portfolios. Second, both the dogmatic and Bayesian investors show high long-horizon correlations between the equity portfolios and the risk-free asset, whereas the agnostic investor perceives these as uncorrelated over all horizons. Mechanically, this is driven by the differences in the loadings from the sorted portfolios on the lagged risk-free returns in the predictive regressions.

[Figure 6 about here.]

5.3 Portfolio choice

Figure 7 shows the optimal allocations for an investor that combines the vulnerable and non-vulnerable sorted portfolios with the risk-free asset. First, we find that the allocation to the risk-free asset is similar to when we combine the risk-free rate with the market portfolio in Figure 4. This is reasonable, given that the loading to equities and the risk-free asset should not be affected too much by the sorting of our portfolios. Together, the vulnerable and non-vulnerable portfolio are still a well-diversified equity portfolio.

Within equities, the agnostic and dogmatic investors present opposite results for the allocation towards the vulnerable and the non-vulnerable portfolios. The agnostic investor generally invests in the non-vulnerable portfolio, because it has higher returns, and buys into the vulnerable portfolio over longer horizons for diversification, when correlations decrease. The dogmatic investor predicts significantly higher returns for the vulnerable portfolio, as is reflected in the allocation. Only at the longest horizons it starts allocating capital towards the non-vulnerable portfolio, because of the increased riskiness of the vulnerable portfolio as documented in Figure 13 in Online Appendix B.2. The dogmatic investor does not combine the two equity portfolios as there is no incentive to diversify with high correlation for all horizons. Finally, the Bayesian investor initially focuses on the vulnerable portfolio, but adds the non-vulnerable portfolio because of diversification benefits at longer horizons, and invests roughly half in each equity portfolio in the long run.

[Figure 7 about here.]

6 Conclusion

We propose a novel approach for measuring the impact of climate change on long-term equity risk and optimal portfolio choice. We characterize the long-horizon dynamics of equity returns by specifying a VAR model that includes the temperature anomaly as a predictor. Because historical data may not be very informative about the impact of climate change on future stock market returns, we estimate the parameters of the VAR using a Bayesian approach that complements historical data with prior information derived from economic theory. Specifically, we elicit prior beliefs from the temperature long-run risk (LRR-T) model of Bansal, Kiku, and Ochoa (2019). Our framework allows for different prior beliefs about the impact of temperature levels on future equity returns. In addition, it explicitly incorporates uncertainty about the impact of climate change in the portfolio optimization.

We document three key findings. First, compared to a benchmark investor with LRR prior beliefs, an investor that takes LRR-T beliefs into account perceives stock markets to be riskier over longer horizons because disasters induced by climate change reduce mean reversion in returns while estimation risk increases. Mean reversion decreases because climate disasters tend to cluster in periods with relatively high temperature levels. In other words, current disasters with an adverse impact on market returns are often quickly followed by new disasters, increasing the correlation between current and future returns. Second, the LRR-T Bayesian investor expects the market risk premium to increase after a climate disaster occurs. In particular, whereas disasters cause a persistent negative shock to risk-free rates, the negative impact on expected market returns is transitory because prices rapidly adjust after a disaster strikes. Therefore, the market risk premium for this investor is higher than that implied by the data and the benchmark Bayesian investor with LRR prior beliefs.

Combined, these first two findings imply that the optimal investor that takes climate change into account through LRR-T prior beliefs buys more equities than the benchmark investor in the short term, because of higher expected market risk premia. For long-term investments, however, increasing temperatures increase the riskiness of the market portfolio, resulting in a larger allocation to the risk-free asset. These differences in the optimal portfolio choice are economically meaningful, with the LRR-T Bayesian investor investing roughly 15 percentage points less in the market than the benchmark LRR Bayesian investor in the long run.

Third, we show that LRR-T prior beliefs have significant impact on investors that assess portfolios vulnerable and non-vulnerable to climate change. In the LRR-T prior, investors are fairly compensated for climate risk, making the vulnerable portfolio the most attractive investment. investors who forecasts solely from historical data expect the non-vulnerable portfolio to outperform and allocate capital to the vulnerable portfolio for diversification purposes. The Bayesian investor initially focuses on the vulnerable portfolio, but invests roughly the same amount in each portfolio in the long run, when the vulnerable and non-vulnerable portfolios becomes less correlated.

Overall, we believe that our paper offers a few important high-level insights into the possible implications of climate change for optimal portfolio allocation by long-term investors such as pension funds and insurance companies. First, we provide a concrete approach to deal with the widely shared concern that using historical data to study portfolio allocation has significant limitations in light of the uncertain long-term effects of climate change not present in the data. Our approach allows investors to combine data with insights from theoretical models and is flexible in the sense that investors can examine the impact of different prior beliefs by varying the weights they place on the data versus the theoretical model. Second, our analysis puts forward three key parameters that investors should consider when studying the impact of climate change on portfolio allocation. In particular, climate change may affect the equity premium, the degree of equity risk over different horizons, as well as the correlations between different asset classes.

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A Appendix

A.1 Solution to the temperature long-run risk model

The intertemporal marginal rate of substitution (IMRS) equals

$$m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{t+1,c}, \tag{12}$$

where $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$.

A.1.1 The wealth-consumption ratio

We use the Euler equation,

$$\mathbb{E}_t[\exp(m_{t+1} + r_{t+1,c})] = 1, \tag{13}$$

to solve the wealth-consumption ratio z_t from Equation (10), filling in the IMRS from Equation (12) and the Campbell-Shiller decomposition for the return on the consumption asset, $r_{t+1,c} = \kappa_0 + \Delta c_{t+1} + \kappa_1 z_{t+1} - z_t$. We find

$$\begin{aligned} m_{t+1} + r_{t+1,c} = {} & \theta \log \delta + \theta \kappa_0 + \theta(\kappa_1 - 1) A_0 + (1 - \gamma + \theta \kappa_1 A_1 \chi \Theta) \mu_c + \theta \kappa_1 A_1 \chi \mu_\varepsilon \\ & + (1 - \gamma + \theta \kappa_1 A_1 \chi \Theta) \sigma_t \eta_{t+1} + \theta \kappa_1 A_1 \chi \sigma_\zeta \zeta_{t+1} + \theta(\kappa_1 \nu - 1) A_3 \sigma_t^2 \\ & + \theta \kappa_1 (1 - \nu) A_3 \bar{\sigma}^2 + \theta \kappa_1 A_3 \sigma_w w_{t+1} + ((1 - \gamma)\rho + \theta(\kappa_1 \rho - 1) A_2) X_t \\ & + (1 - \gamma + \theta \kappa_1 A_2) d \Delta N_{t+1} + \theta(\kappa_1 \nu_\varepsilon - 1) A_1 T_t. \end{aligned} \tag{14}$$

With the expectation

$$\begin{aligned} \mathbb{E}_t[m_{t+1} + r_{t+1,c}] = {} & \theta \log \delta + \theta \kappa_0 + \theta(\kappa_1 - 1) A_0 + (1 - \gamma + \theta \kappa_1 A_1 \chi \Theta) \mu_c + \theta \kappa_1 A_1 \chi \mu_\varepsilon \\ & + \theta(\kappa_1 \nu - 1) A_3 \sigma_t^2 + \theta \kappa_1 (1 - \nu) A_3 \bar{\sigma}^2 + ((1 - \gamma)\rho + \theta(\kappa_1 \rho - 1) A_2) X_t \\ & + (1 - \gamma + \theta \kappa_1 A_2) d \lambda_0 \Delta t + ((1 - \gamma + \theta \kappa_1 A_2) d \lambda_1 \Delta t + \theta(\kappa_1 \nu_\varepsilon - 1) A_1) T_t \end{aligned} \tag{15}$$

and variance

$$\begin{aligned}\mathrm{Var}_t(m_{t+1}+r_{t+1,c}) = {} & (1-\gamma+\theta\kappa_1 A_1\chi\Theta)^2\sigma_t^2 + (\theta\kappa_1 A_1\chi)^2\sigma_\zeta^2 + (\theta\kappa_1 A_3)^2\sigma_w^2 \\ & + (1-\gamma+\theta\kappa_1 A_2)^2 d^2\lambda_0\Delta t + (1-\gamma+\theta\kappa_1 A_2)^2 d^2\lambda_1\Delta t T_t.\end{aligned} \tag{16}$$

We now go back to the Euler equation from (13) and take logs and use Jensen's inequality to find the solution to the price-consumption ratio from Equation (10):

$$\begin{aligned}\theta(1-\kappa_1\nu)A_3 &= 0.5(1-\gamma+\theta\kappa_1 A_1\chi\Theta)^2, \\ \theta(1-\kappa_1\rho)A_2 &= (1-\gamma)\rho, \\ \theta(1-\kappa_1\nu_\varepsilon)A_1 &= (1-\gamma+\theta\kappa_1 A_2)d\left(1+\frac{(1-\gamma+\theta\kappa_1 A_2)d}{2}\right)\lambda_1\Delta t, \\ (1-\kappa_1)A_0 &= \log\delta+\kappa_0+\left(1-\frac{1}{\psi}+\kappa_1 A_1\chi\Theta\right)\mu_c+\kappa_1 A_1\chi\mu_\varepsilon \\ & \quad + \kappa_1(1-\nu)A_3\bar{\sigma}^2 + 0.5\theta((\kappa_1 A_1\chi)^2\sigma_\zeta^2+(\kappa_1 A_3)^2\sigma_w^2) \\ & \quad + \left(1-\frac{1}{\psi}+\kappa_1 A_2\right)d\left(1+\frac{(1-\gamma+\theta\kappa_1 A_2)d}{2}\right)\Delta t\lambda_0.\end{aligned} \tag{17}$$

A.1.2 The risk-free rate

Again, we use the Euler equation to solve the risk-free rate. We start from the IMRS of Equation (12) and fill in

$$\begin{aligned}m_{t+1} = {} & \theta\log\delta+(\theta-1)\kappa_0+(\theta-1)(\kappa_1-1)A_0+((\theta-1)\kappa_1 A_1\chi\Theta-\gamma)\mu_c+(\theta-1)\kappa_1 A_1\chi\mu_\varepsilon \\ & + (\theta-1)\kappa_1 A_3(1-\nu)\bar{\sigma}^2+(\theta-1)\kappa_1 A_1\chi\sigma_\zeta\zeta_{t+1}+(\theta-1)\kappa_1 A_3\sigma_w w_{t+1} \\ & + ((\theta-1)\kappa_1 A_1\chi\Theta-\gamma)\sigma_t\eta_{t+1}+(\theta-1)(\kappa_1\nu-1)A_3\sigma_t^2 \\ & + ((\theta-1)(\kappa_1\rho-1)A_2-\gamma\rho)X_t+((\theta-1)\kappa_1 A_2-\gamma)d\Delta N_{t+1}+(\theta-1)(\kappa_1\nu_\varepsilon-1)A_1 T_t.\end{aligned} \tag{18}$$

We then have expectation

$$
\begin{aligned}
\mathbb{E}_t[m_{t+1}+r_{f,t}] = {} & r_{f,t}+\theta\log\delta+(\theta-1)\kappa_0+(\theta-1)(\kappa_1-1)A_0+((\theta-1)\kappa_1 A_1\chi\Theta-\gamma)\mu_c \\
& +(\theta-1)\kappa_1 A_1\chi\mu_\varepsilon+(\theta-1)\kappa_1 A_3(1-\nu)\bar{\sigma}^2+(\theta-1)(\kappa_1\nu-1)A_3\sigma_t^2 \\
& +((\theta-1)(\kappa_1\rho-1)A_2-\gamma\rho)X_t+((\theta-1)\kappa_1 A_2-\gamma)d\lambda_0\Delta t \\
& +(((\theta-1)\kappa_1 A_2-\gamma)d\lambda_1\Delta t+(\theta-1)(\kappa_1\nu_\varepsilon-1)A_1)T_t,
\end{aligned}
\tag{19}
$$

and variance

$$
\begin{aligned}
\mathrm{Var}_t(m_{t+1}+r_{f,t}) = {} & ((\theta-1)\kappa_1 A_1\chi)^2\sigma_\zeta^2+((\theta-1)\kappa_1 A_3)^2\sigma_w^2+((\theta-1)\kappa_1 A_1\chi\Theta-\gamma)^2\sigma_t^2 \\
& +((\theta-1)\kappa_1 A_2-\gamma)^2 d^2\lambda_0\Delta t+((\theta-1)\kappa_1 A_2-\gamma)^2 d^2\lambda_1\Delta t T_t.
\end{aligned}
\tag{20}
$$

With the Euler equation, logs, and Jensen's inequality we find

$$
\begin{aligned}
0 = {} & \mathbb{E}_t[m_{t+1}+r_{f,t}]+0.5\mathrm{Var}_t(m_{t+1}+r_{f,t}) \\
\Leftrightarrow r_{f,t} = {} & r_f-((\theta-1)(\kappa_1\rho-1)A_2-\gamma\rho)X_t \\
& -\left(((\theta-1)\kappa_1 A_2-\gamma)d\left(1+\frac{((\theta-1)\kappa_1 A_2-\gamma)d}{2}\right)\lambda_1\Delta t+(\theta-1)(\kappa_1\nu_\varepsilon-1)A_1\right)T_t \\
& -((\theta-1)(\kappa_1\nu-1)A_3+0.5((\theta-1)\kappa_1 A_1\chi\Theta-\gamma)^2)\sigma_t^2, \\
r_f = {} & -\theta\log\delta+\gamma\mu_c-(\theta-1)(\kappa_0+(\kappa_1-1)A_0+\kappa_1 A_1\chi\Theta\mu_c) \\
& -(\theta-1)(\kappa_1 A_1\chi\mu_\varepsilon+\kappa_1 A_3(1-\nu)\bar{\sigma}^2) \\
& -0.5[((\theta-1)\kappa_1 A_1\chi)^2\sigma_\zeta^2+((\theta-1)\kappa_1 A_3)^2\sigma_w^2] \\
& -((\theta-1)\kappa_1 A_2-\gamma)d\left(1+\frac{((\theta-1)\kappa_1 A_2-\gamma)d}{2}\right)\Delta t\lambda_0.
\end{aligned}
\tag{21}
$$

A.1.3 The price-dividend ratio

We similarly solve the price-dividend ratio of portfolio i, $z_{t,i}$ from Equation (10). The Campbell-Shiller decomposition for the returns of portfolio i gives us

$$\begin{aligned} r_{t+1,i} &= \kappa_{0,i} + \Delta d_{t+1,i} + \kappa_{1,i} z_{t+1,i} - z_{t,i} \\ &= \kappa_{0,i} + \mu_d + (\kappa_{1,i} - 1)A_{0,i} + \kappa_{1,i}A_{1,i}\chi\mu_\varepsilon + \kappa_{1,i}A_{1,i}\chi\Theta\mu_c \\ &\quad + \kappa_{1,i}A_{3,i}(1-\nu)\bar{\sigma}^2 + \varphi_d\sigma_t u_{t+1} + (\pi_d + \kappa_{1,i}A_{1,i}\chi\Theta)\sigma_t\eta_{t+1} + \kappa_{1,i}A_{1,i}\chi\sigma_\zeta\zeta_{t+1} \\ &\quad + \kappa_{1,i}A_{3,i}\sigma_w w_{t+1} + (\kappa_{1,i}\nu_\varepsilon - 1)A_{1,i}T_t + (\phi_i\rho + (\kappa_{1,i}\rho - 1)A_{2,i})X_t \\ &\quad + (\phi_i + \kappa_{1,i}A_{2,i})d\Delta N_{t+1} + A_{3,i}(\kappa_{1,i}\nu - 1)\sigma_t^2 \end{aligned} \tag{22}$$

Combining this with the IMRS from Equation (18) we find the expectation

$$\begin{aligned} \mathbb{E}_t[m_{t+1} + r_{t+1,i}] &= \theta\log\delta + (\theta-1)\kappa_0 + \kappa_{0,i} + (\theta-1)(\kappa_1-1)A_0 + (\kappa_{1,i}-1)A_{0,i} + \mu_d \\ &\quad + (((\theta-1)\kappa_1A_1 + \kappa_{1,i}A_{1,i})\chi\Theta - \gamma)\mu_c + ((\theta-1)\kappa_1A_1 + \kappa_{1,i}A_{1,i})\chi\mu_\varepsilon \\ &\quad + ((\theta-1)\kappa_1A_3 + \kappa_{1,i}A_{3,i})(1-\nu)\bar{\sigma}^2 \\ &\quad + ((\theta-1)(\kappa_1\nu-1)A_3 + (\kappa_{1,i}\nu-1)A_{3,i})\sigma_t^2 \\ &\quad + ((\theta-1)(\kappa_1\rho-1)A_2 + (\kappa_{1,i}\rho-1)A_{2,i} + \phi_i\rho - \gamma\rho)X_t \\ &\quad + [(\theta-1)(\kappa_1\nu_\varepsilon-1)A_1 + (\kappa_{1,i}\nu_\varepsilon-1)A_{1,i} \\ &\quad + ((\theta-1)\kappa_1A_2 + \kappa_{1,i}A_{2,i} + \phi_i - \gamma)d\lambda_1\Delta t]T_t \\ &\quad + ((\theta-1)\kappa_1A_2 + \kappa_{1,i}A_{2,i} + \phi_i - \gamma)d\lambda_0\Delta t, \end{aligned} \tag{23}$$

and variance

$$\begin{aligned} \mathrm{Var}_t(m_{t+1} + r_{t+1,i}) &= ((\theta-1)\kappa_1A_3 + \kappa_{1,i}A_{3,i})^2\sigma_w^2 \\ &\quad + ((\theta-1)\kappa_1A_1 + \kappa_{1,i}A_{1,i})^2\chi^2\sigma_\zeta^2 \\ &\quad + \varphi_d^2\sigma_t^2 + (((\theta-1)\kappa_1A_1 + \kappa_{1,i}A_{1,i})\chi\Theta + \pi_d - \gamma)^2\sigma_t^2 \\ &\quad + ((\theta-1)\kappa_1A_2 + \kappa_{1,i}A_{2,i} + \phi_i - \gamma)^2d^2\lambda_0\Delta t \\ &\quad + ((\theta-1)\kappa_1A_2 + \kappa_{1,i}A_{2,i} + \phi_i - \gamma)^2d^2\lambda_1\Delta t T_t. \end{aligned} \tag{24}$$

Which we fill in in the Euler equation and take logs to find

$$
\begin{aligned}
0 &= \log \mathbb{E}_t[\exp(m_{t+1} + r_{t+1,i})] = \mathbb{E}_t[(m_{t+1} + r_{t+1,i})] + 0.5\mathrm{Var}_t(m_{t+1} + r_{t+1,i}) \\
&= \theta \log \delta + (\theta - 1)\kappa_0 + \kappa_{0,i} + (\theta - 1)(\kappa_1 - 1)A_0 + (\kappa_{1,i} - 1)A_{0,i} + \mu_d \\
&\quad + (((\theta - 1)\kappa_1 A_1 + \kappa_{1,i} A_{1,i})\chi\Theta - \gamma)\mu_c + ((\theta - 1)\kappa_1 A_1 + \kappa_{1,i} A_{1,i})\chi\mu_\varepsilon \\
&\quad + C_N(1 + 0.5C_N)\lambda_0 \Delta t + ((\theta - 1)\kappa_1 A_3 + \kappa_{1,i} A_{3,i})(1 - \nu)\bar{\sigma}^2 \\
&\quad + 0.5(((\theta - 1)\kappa_1 A_3 + \kappa_{1,i} A_{3,i})^2 \sigma_w^2 + ((\theta - 1)\kappa_1 A_1 + \kappa_{1,i} A_{1,i})^2 \chi^2 \sigma_\zeta^2) \\
&\quad + [(\theta - 1)(\kappa_1 \nu - 1)A_3 + (\kappa_{1,i}\nu - 1)A_{3,i} \\
&\quad + 0.5\varphi_d^2 + 0.5(((\theta - 1)\kappa_1 A_1 + \kappa_{1,i} A_{1,i})\chi\Theta + \pi_d - \gamma)^2]\sigma_t^2 \\
&\quad + ((\theta - 1)(\kappa_1 \rho - 1)A_2 + (\kappa_{1,i}\rho - 1)A_{2,i} + \phi_i \rho - \gamma\rho)X_t \\
&\quad + ((\theta - 1)(\kappa_1 \nu_\varepsilon - 1)A_1 + (\kappa_{1,i}\nu_\varepsilon - 1)A_{1,i} + C_N(1 + 0.5C_N)\lambda_1 \Delta t)T_t, \\
C_N &= ((\theta - 1)\kappa_1 A_2 + \kappa_{1,i} A_{2,i} + \phi_i - \gamma)d.
\end{aligned}
\tag{25}
$$

We set all terms in front of T_t, X_t and σ_t^2 to zero to solve for $z_{t,i}$:

$$
\begin{aligned}
(1 - \kappa_{1,i}\nu)A_{3,i} &= 0.5(((\theta - 1)\kappa_1 A_1 + \kappa_{1,i} A_{1,i})\chi\Theta + \pi_d - \gamma)^2 \\
&\quad + 0.5\varphi_d^2 + (\theta - 1)(\kappa_1 \nu - 1)A_3, \\
(1 - \kappa_{1,i}\rho)A_{2,i} &= (\theta - 1)(\kappa_1 \rho - 1)A_2 + \phi_i \rho - \gamma\rho, \\
(1 - \kappa_{1,i}\nu_\varepsilon)A_{1,i} &= (\theta - 1)(\kappa_1 \nu_\varepsilon - 1)A_1 + C_N(1 + 0.5C_N)\lambda_1 \Delta t, \\
(1 - \kappa_{1,i})A_{0,i} &= \theta \log \delta + (\theta - 1)\kappa_0 + \kappa_{0,i} + (\theta - 1)(\kappa_1 - 1)A_0 + \mu_d \\
&\quad + (((\theta - 1)\kappa_1 A_1 + \kappa_{1,i} A_{1,i})\chi\Theta - \gamma)\mu_c + ((\theta - 1)\kappa_1 A_1 + \kappa_{1,i} A_{1,i})\chi\mu_\varepsilon \\
&\quad + C_N(1 + 0.5C_N)\lambda_0 \Delta t + ((\theta - 1)\kappa_1 A_3 + \kappa_{1,i} A_{3,i})(1 - \nu)\bar{\sigma}^2 \\
&\quad + 0.5(((\theta - 1)\kappa_1 A_3 + \kappa_{1,i} A_{3,i})^2 \sigma_w^2 + ((\theta - 1)\kappa_1 A_1 + \kappa_{1,i} A_{1,i})^2 \chi^2 \sigma_\zeta^2), \\
C_N &= ((\theta - 1)\kappa_1 A_2 + \kappa_{1,i} A_{2,i} + \phi_i - \gamma)d.
\end{aligned}
\tag{26}
$$

A.1.4 The equity risk premium

As discussed by Bansal and Yaron (2004), the risk premium of any asset is based on the covariance between the unexpected returns and innovations in the SDF m_{t+1} as

$$\ln\mathbb{E}_t[R_{i,t+1}] - r_{f,t} = \mathbb{E}_t[r_{i,t+1} - r_{f,t}] + 0.5\text{Var}_t(r_{i,t+1}) = -\text{Cov}_t(m_{t+1} - \mathbb{E}_t[m_{t+1}], r_{i,t+1} - \mathbb{E}_t[r_{i,t+1}]). \tag{27}$$

The innovation in the SDF based on Equation (12) equals

$$m_{t+1} - \mathbb{E}_t[m_{t+1}] = -\lambda_\eta \sigma_t \eta_{t+1} - \lambda_\zeta \sigma_\zeta \zeta_{t+1} - \lambda_w \sigma_w w_{t+1} - \lambda_X(\Delta N_{t+1} - (\lambda_0 \Delta t + \lambda_1 \Delta t T_t)), \tag{28}$$

where

$$\begin{aligned} \lambda_\eta &= \gamma + (1-\theta)\kappa_1 A_1 \chi \Theta, \\ \lambda_\zeta &= (1-\theta)\kappa_1 A_1 \chi, \\ \lambda_w &= (1-\theta)\kappa_1 A_3, \\ \lambda_X &= (\gamma + (1-\theta)\kappa_1 A_2) d. \end{aligned} \tag{29}$$

Based on Equation (22), the innovation in the return of portfolio i is given by

$$\begin{aligned} r_{i,t+1} - \mathbb{E}_t[r_{i,t+1}] = {} & \varphi_d \sigma_t u_{t+1} + (\pi_d + \kappa_{1,i} A_{1,i} \chi \Theta) \sigma_t \eta_{t+1} + \kappa_{1,i} A_{1,i} \chi \sigma_\zeta \zeta_{t+1} \\ & + \kappa_{1,i} A_{3,i} \sigma_w w_{t+1} + (\phi_i + \kappa_{1,i} A_{2,i}) d(\Delta N_{t+1} - (\lambda_0 \Delta t + \lambda_1 \Delta t T_t)) \end{aligned} \tag{30}$$

This brings us to the conditional risk premium of asset i:

$$\begin{aligned} \ln\mathbb{E}_t[R_{i,t+1}] - r_{f,t} &= -\text{Cov}_t(m_{t+1} - \mathbb{E}_t[m_{t+1}], r_{i,t+1} - \mathbb{E}_t[r_{i,t+1}]) \\ &= \lambda_\eta \beta_{i,\eta} \sigma_t^2 + \lambda_\zeta \beta_{i,\zeta} \sigma_\zeta^2 + \lambda_w \beta_{i,w} \sigma_w^2 + \lambda_X \beta_{i,X} (\lambda_0 \Delta t + \lambda_1 \Delta t T_t), \end{aligned} \tag{31}$$

where lambdas are given in Equation (29) and betas are

$$\begin{aligned}
\beta_{i,\eta} &= \pi_d + \kappa_{1,i} A_{1,i} \chi \Theta, \\
\beta_{i,\zeta} &= \kappa_{1,i} A_{1,i} \chi, \\
\beta_{i,w} &= \kappa_{1,i} A_{3,i}, \\
\beta_{i,X} &= (\phi_i + \kappa_{1,i} A_{2,i}) d.
\end{aligned} \tag{32}$$

A.1.5 Solving the fixed-point problem

The approximation constants $\kappa_{0(,i)}$ and $\kappa_{1(,i)}$ are defined as

$$\begin{aligned}
\kappa_{0(,i)} &= \log(1 + \exp(\bar{z}_{(i)})) - \kappa_{1(,i)} \bar{z}_{(i)}, \\
\kappa_{1(,i)} &= \frac{\exp(\bar{z}_{(i)})}{1 + \exp(\bar{z}_{(i)})},
\end{aligned} \tag{33}$$

where $\bar{z}_{(i)}$ is the mean of the wealth-consumption ratio z_t or price-dividend ratio of portfolio i, $z_{t,i}$. We solve the mean of these ratios by numerically (through iteration) solving the fixed point problem

$$\begin{aligned}
\bar{z}_{(i)} &= A_{0(,i)} + A_{1(,i)} \bar{T}_t + A_{2(,i)} \bar{X}_t + A_{3(,i)} \bar{\sigma}_t^2 \\
&= A_{0(,i)} + A_{1(,i)} \left(\frac{1}{n} \mathbb{E}_0 \left[\sum_{t=1}^{n} T_t \right] \right) + A_{2(,i)} \left(\frac{1}{n} \mathbb{E}_0 \left[\sum_{t=1}^{n} X_t \right] \right) + A_{3(,i)} \bar{\sigma_t}^2,
\end{aligned} \tag{34}$$

where the expected averages $\bar{T}_t$ and $\bar{X}_t$ over the sample from period $t = 1$ to $t = n$ are given as

$$\begin{aligned}
\frac{1}{n} \mathbb{E}_0 \left[\sum_{t=1}^{n} T_t \right] &= \left(\frac{T_0}{n} - \frac{\chi(\mu_\varepsilon + \Theta \mu_c)}{n(1-\nu_\varepsilon)} \right) \left(\frac{\nu_\varepsilon - \nu_\varepsilon^{n+1}}{1-\nu_\varepsilon} \right) + \frac{\chi(\mu_\varepsilon + \Theta \mu_c)}{1-\nu_\varepsilon}, \\
\frac{1}{n} \mathbb{E}_0 \left[\sum_{t=1}^{n} X_t \right] &= \frac{1}{n} \left(-X_0 + \sum_{t=0}^{n} \mathbb{E}_0[X_t] \right) \\
&= \frac{X_0}{n} \left(\frac{\rho - \rho^{n+1}}{1-\rho} \right) + \frac{1}{n} \left(\frac{d \Delta t \lambda_0}{1-\rho} + \frac{d \Delta t \lambda_1}{1-\rho} \frac{\chi(\mu_\varepsilon + \Theta \mu_c)}{1-\nu_\varepsilon} \right) \left(n - \frac{\rho - \rho^{n+1}}{1-\rho} \right) \\
&\quad + \frac{1}{n \nu_\varepsilon} \left(\frac{d \Delta t \lambda_1}{1 - \left(\frac{\rho}{\nu_\varepsilon} \right)} \right) \left(T_0 - \frac{\chi(\mu_\varepsilon + \Theta \mu_c)}{1-\nu_\varepsilon} \right) \left(\frac{1-\nu_\varepsilon^{n+1}}{1-\nu_\varepsilon} - \frac{1-\rho^{n+1}}{1-\rho} \right).
\end{aligned} \tag{35}$$

The sample used in our regressions starts in 1947Q1 and runs until 2019Q4, 292 observations of quarterly data. In our simulations, we construct a similar sample. We simulate $n = 939$ months of data, resulting in a sample of 292 quarters after we drop the first five years of our simulation as burn-in and the last three months because of a lead in the risk-free yields. We calibrate the model to set the expected starting temperature in our simulations to zero, i.e. $\mathbb{E}_0[T_{60}] = 0$, based on our burn-in of 60 months. The risk-free yields are led by one period, since the assumed absence of credit risk implies that the risk-free return realized at the end of quarter Q_i is already observed in Q_{i-1}.

A.2 Adjustments to the LRR-T model

The LRR-T model presented in Section 3.1 deviates from the LRR-T model of Bansal, Kiku, and Ochoa (2019) in several ways.

First, in Equation (7), the dividend growth process allows for different loadings on the different shocks in the model and we let volatility vary over time. Both adjustments increase the flexibility of the model and are generally implemented in recent versions of LRR model (among others, in Bansal, Kiku, and Yaron (2012)). We allow for portfolio-specific disasters impact through a portfolio-specific ϕ_i.

Second, in Equation (8), we include a persistence $\rho < 1$ of the economic impact of disasters X, instead of $\rho = 1$. We believe that it is reasonable that climate-related disasters have a persistent impact on consumption and dividend growth, but that impact should not be indefinite. In the same equation, we let the increments of the disaster process (ΔN) be Poisson distributed, instead of the whole process N. With this adjustment, we make the current disaster intensity dependent on recent temperature levels, instead of on the historical path of temperature growth.

Third, in Equation (9), the atmospheric carbon concentration includes a separate trend μ_ε. This trend is used to generate the different climate scenarios in our analysis.

Finally, in Equation (10), we assume that the log wealth-consumption ratio z_t and the log price-dividend ratio $z_{t,m}$ also depend on the economic disaster impact X_t and time-varying variance σ_t^2. The inclusion of σ_t^2 follows the inclusion of time-varying volatility in Equation (7). We believe that the the inclusion of economic disaster impact X_t in the processes for z_t and $z_{t,m}$ is intuitively reasonable. When temperatures increase, z_t and $z_{t,m}$ should decrease as prices decrease to adjust for expected future disasters. Without sufficient precautionary savings, these ratios should also be affected when disasters occur, because these disasters should have significant cashflow impact.

A.3 Posterior VAR distributions

We follow Avramov, Cederburg, and Lucivjanska (2018) and estimate the VAR model in Equation (1) as

$$\begin{bmatrix} r_{m,t+1} \\ p_{t+1} - d_{t+1} \\ r_{f,t} \\ T_{t+1} \end{bmatrix} = C' \begin{bmatrix} 1 \\ p_t - p_d \\ r_{f,t-1} \\ T_t \end{bmatrix} + \epsilon_{t+1}, \qquad \epsilon_{t+1} \sim N(0, \Sigma). \tag{36}$$

where C is the VAR coefficients matrix and Σ is the variance-covariance matrix of the residuals. As shown in the Appendix of Avramov, Cederburg, and Lucivjanska (2018), the posterior distribution of the VAR parameters with the model-based prior conditional on the observed data for periods $1, \ldots, t$, D_t, and the asset pricing model parameters Θ_M is given by

$$\begin{aligned} \Sigma \mid D_t, \Theta_M &\sim IW((\omega_M + \omega_D)N\hat{\Sigma}(\Theta_M), (\omega_M + \omega_D)N - 4), \\ C \mid \Sigma, D_t, \Theta_M &\sim N(\hat{C}(\Theta_M), \Sigma \otimes (\omega_M N \Gamma^*_{xx} + \omega_D X'X)^{-1}), \end{aligned} \tag{37}$$

in which

$$\begin{aligned} \hat{\Sigma}(\Theta_M) = \frac{1}{(\omega_M + \omega_D)N} &[(\omega_M N \Gamma^*_{yy} + \omega_D Y'Y) \\ &- (\omega_M N \Gamma^*_{xy} + \omega_D Y'X)(\omega_M N \Gamma^*_{xx} + \omega_D X'X)^{-1}(\omega_M N \Gamma^*_{xy} + \omega_D X'Y)], \\ \hat{C}(\Theta_M) = (\omega_M N \Gamma^*_{xx} &+ \omega_D X'X)^{-1}(\omega_M N \Gamma^*_{xy} + \omega_D X'Y), \end{aligned} \tag{38}$$

N is the number of observations in our sample (292 quarters), and Γ^*_{xx}, Γ^*_{xy}, and Γ^*_{yy} are the population moments from the asset pricing models in Section 3.1, conditional on the model parameters Θ_M, as introduced in the Appendix of Avramov, Cederburg, and Lucivjanska (2018).

Finally, $\frac{\omega_M}{\omega_M+\omega_D}$ and $\frac{\omega_D}{\omega_M+\omega_D}$ are the weight given to the model-based prior and the historical data, respectively. For the Bayesian investor, we set equal weights to the informative prior and historical data by setting $\omega_M = \omega_D = 1$. For the agnostic investor we specify $\omega_M = 0$ and $\omega_D = 1$, and Equation (37) then equals the posterior of the analysis on historical data with a multivariate Jeffreys uninformative prior. For the climate change believer we specify $\omega_M = 1$ and $\omega_D = 0$, and Equation (37) then equals the prior distribution solely based on an asset pricing model.

B Online Appendix

B.1 Numerical optimal asset allocation

We numerically solve the optimal asset allocation for our buy-and-hold long-only investor following the methodology from, among others, Barberis (2000). This appendix explains our methodology. We closely follow a similar explanation in the appendix from Diris (2014) in this section.

Sampling from the predictive distribution

We sample $N = 250,000$ paths of length $K = 100$ quarters from the predictive distribution of the asset returns and state variables in our VAR model. We repeat the following two steps N times:

1. For k in $1, \ldots, K$, Sample the returns and state variables in period k conditional on the posterior mean VAR parameters (a, B, Σ) and the predictor variable values in $k-1$. We start forecasting from the last observed value of our predictor variables in the data at period $k = 1$.

2. Re-sample the asset returns and predictor variables in period k when we draw a quarterly return for the risk-free asset below -10%. This step is needed to avoid minus infinity utility, as an investor can never go bankrupt as long as it invests in the risk-free asset with this restriction. In practice, re-sampling is hardly ever required.

Calculation of buy-and-hold portfolio

Based on the N return forecasts from above we now follow the steps below to compute the optimal investment portfolio.

1. Make a grid of portfolio weights. We invest long only, i.e. weights are between zero and one, and use steps of 0.01 for our grid search. Then, for each weight in the grid:

2. Take a set of weights and calculate realized utility for each simulated path.

3. Approximate expected utility with the mean of these N realized utilities.

For each horizon $k = 1, \ldots, K$, we choose the optimal vector of weights as the one that results in the largest expected utility from 3.

B.2 Extra output

[Table 6 about here.]

[Figure 8 about here.]

[Figure 9 about here.]

[Figure 10 about here.]

[Figure 11 about here.]

[Figure 12 about here.]

[Figure 13 about here.]

Table 1: Calibration parameters for the temperature long-run risk model

Preferences	δ	γ	ψ					
	0.998	5	1.5					
Consumption	μ_c	ρ	d	$\bar{\sigma}$	ν	σ_w		
	0.0052	0.99	-0.004166	0.0072	0.999	0.0000028		
Dividend	μ_d	π_d	ϕ_m	ϕ_v	ϕ_{nv}	φ_d		
	0.0062	2.0	1.1	1.35	0.85	5.0		
Climate	ν_e	ε_0	μ_e	Θ	σ_ζ	χ	λ_0	λ_1
	0.9971	-1	0.0095	1	0.5	0.2	0.075	0.075

This table shows the calibration parameters for the temperature long-run risk model from Equations (7) to (10), based on a monthly decision interval. This calibration matches historically observed climate change, with the temperature anomaly increasing from 0 to 1 degrees Celsius in expectation in our simulations. ϕ_m, ϕ_v and ϕ_{nv} are the parameters for the different growth processes of the market portfolio and the portfolios vulnerable and non-vulnerable to temperature innovations, respectively.

Table 2: Quarterly calibration moments for the (temperature) long-run risk models

	Data	LRR	LRR-T
$E(r_m)$	1.81	1.78	1.71
$\sigma(r_m)$	7.25	8.01	8.20
$E(r_f)$	0.13	0.20	0.17
$\sigma(r_f)$	0.47	0.28	0.62
$E(p-d)$	4.89	4.45	4.42
$\sigma(p-d)$	0.44	0.16	0.23
$E(\Delta c)$	0.47	0.45	0.45
$\sigma(\Delta c)$	0.49	1.14	1.43
$E(\Delta d)$	0.66	0.64	0.62
$\sigma(\Delta d)$	1.94	5.83	5.92
$E(T)$	0.33		0.64
$\sigma(T)$	0.83		0.83

This table reports the first and second moments of market returns, ex-ante risk-free yields, price-dividend ratios, consumption and dividend growth, and the temperature anomaly. Returns and growth rates are in percentages and all moments except for the temperature anomaly are in logs. The first column reports the historical moments from monthly data time-aggregated to quarterly values from 1947Q1 to 2019Q4, for all series except for consumption growth. For consumption growth, these moments are based on quarterly data from 1947Q2 to 2019Q4. The columns on the right show the population moments from 250,000 simulations from the LRR(-T) models, each simulation matching the 1947Q1-2019Q4 sample.

Table 3: VAR parameter estimates for the agnostic, dogmatic and Bayesian investors

Panel A: Data (uninformative prior) - Agnostic

	Intercept	$p_t - d_t$	$r_{f,t}$	T_t
$r_{m,t+1}$	14.28	-2.63	80.55	0.89
	(2.76)	(-2.46)	(0.87)	(1.58)
$p_{t+1} - d_{t+1}$	0.10	0.98	1.37	0.01
	(1.79)	(89.10)	(1.44)	(1.22)
$r_{f,t+1}$	0.19	-0.03	88.56	0.01
	(1.54)	(-1.36)	(40.83)	(0.50)
T_{t+1}	-2.48	0.57	-11.36	0.19
	(-4.65)	(5.13)	(-1.18)	(3.23)

Panel B: LRR-T prior - Dogmatic

	Intercept	$p_t - d_t$	$r_{f,t}$	T_t
$r_{m,t+1}$	32.05	-6.87	273.50	-0.61
	(1.18)	(-1.14)	(2.29)	(-0.49)
$p_{t+1} - d_{t+1}$	0.93	0.79	3.31	-0.04
	(4.86)	(18.78)	(3.93)	(-4.09)
$r_{f,t+1}$	0.36	-0.08	99.05	-0.03
	(0.90)	(-0.88)	(56.52)	(-1.48)
T_{t+1}	0.42	-0.09	0.97	0.98
	(0.92)	(-0.90)	(0.47)	(46.25)

Panel C: Data with LRR-T prior - Bayesian

	Intercept	$p_t - d_t$	$r_{f,t}$	T_t
$r_{m,t+1}$	7.11	-1.23	155.94	0.33
	(1.90)	(-1.54)	(2.75)	(0.90)
$p_{t+1} - d_{t+1}$	0.05	0.99	0.76	0.00
	(1.58)	(141.86)	(1.53)	(0.71)
$r_{f,t+1}$	-0.01	0.01	95.37	-0.02
	(-0.16)	(0.38)	(87.39)	(-2.34)
T_{t+1}	0.74	-0.12	-20.65	0.70
	(2.44)	(-1.87)	(-4.43)	(23.90)

Panel D: Data with LRR prior - Bayesian

	Intercept	$p_t - d_t$	$r_{f,t}$
$r_{m,t+1}$	8.22	-1.43	148.17
	(2.26)	(-1.82)	(1.84)
$p_{t+1} - d_{t+1}$	0.05	0.99	1.22
	(1.75)	(146.40)	(1.75)
$r_{f,t+1}$	0.04	0.00	91.30
	(0.57)	(-0.30)	(64.61)

This table shows the posterior means of the parameter estimates of VAR from Equation (1). Posterior t-stats are reported below the coefficients. Panel A reports the VAR for the agnostic investor, based on uninformative priors. Panel B reports the VAR for the dogmatic climate risk believer, based on the LRR-T prior. Panel C reports the VAR for the LRR-T Bayesian investor that combines data with the baseline LRR-T prior. Panel D is the benchmark LRR Bayesian investor. The regressions are based on a quarterly sample from 1947Q1 to 2019Q4. Returns are in log percentages, the price-dividend ratio is in logs and the temperature anomaly is in degrees Celsius.

Table 4: Descriptive statistics of the quarterly returns on the vulnerable and non-vulnerable portfolios.

Portfolio	Mean	StDev	Min	Max	Skew	Kurt	Sharpe	Corr
Non-Vul	1.96	10.32	-36.20	34.15	-0.72	4.83	0.18	0.84
Vul	1.67	8.94	-28.85	21.51	-0.78	3.93	0.17	

This table shows summary statistics of the vulnerable and non-vulnerable quarterly returns from 1947Q1 to 2019Q4. Returns are in log percentages.

Table 5: Predictive regressions for (non-)vulnerable portfolio returns

Panel A: Data (uninformative prior) - Agnostic				
	Intercept	$p_t - d_t$	$r_{f,t}$	T_t
$r_{v,t+1}$	15.28	-2.84	37.29	0.65
	(2.44)	(-2.19)	(0.33)	(0.94)
$r_{nv,t+1}$	15.97	-2.92	12.53	0.81
	(2.20)	(-1.95)	(0.10)	(1.01)
Panel B: LRR-T prior - Dogmatic				
	Intercept	$p_t - d_t$	$r_{f,t}$	T_t
$r_{v,t+1}$	41.09	-8.84	326.90	-0.75
	(1.47)	(-1.42)	(2.64)	(-0.58)
$r_{nv,t+1}$	21.07	-4.51	208.80	-0.46
	(0.83)	(-0.80)	(1.85)	(-0.40)
Panel C: Data with LRR-T prior - Bayesian				
	Intercept	$p_t - d_t$	$r_{f,t}$	T_t
$r_{v,t+1}$	9.88	-1.81	159.25	0.28
	(2.38)	(-2.04)	(2.47)	(0.68)
$r_{nv,t+1}$	4.94	-0.76	105.70	0.04
	(1.13)	(-0.82)	(1.57)	(0.08)

This table shows the posterior means of the parameter estimates of the predictive regression from the returns of the vulnerable and non-vulnerable portfolios on the lagged price-dividend ratio, risk-free return and temperature anomaly. Posterior t-stats are reported below the coefficients. Panel A reports the coefficient estimates for the agnostic investor, based on uninformative priors. Panel B reports the coefficient estimates for the dogmatic climate risk believer, based on the LRR-T prior. Panel C reports the coefficient estimates for the LRR-T Bayesian investor that combines data with the LRR-T prior. The regressions are based on a quarterly sample from 1947Q1 to 2019Q4. Returns are in log percentages, the price-dividend ratio is in logs and the temperature anomaly is in degrees Celsius.

Table 6: Population estimates for VAR parameters from the temperature long-run risk model

	Intercept	$p_t - d_t$	$r_{f,t}$	T_t
$r_{m,t+1}$	32.07	-6.87	273.55	-0.61
$p_{t+1} - d_{t+1}$	0.93	0.79	3.32	-0.04
$r_{f,t+1}$	0.36	-0.08	99.04	-0.03
T_{t+1}	0.43	-0.09	1.01	0.98

This table shows the model coefficients of the VAR from Equation (1) estimated on a very large set of 250,000 simulations from the LRR-T model. The model is estimated on a quarterly time interval and each simulation matches the 1947Q1-2019Q4 sample period. Returns are in log percentages, the price-dividend ratio is in logs and the temperature anomaly is in degrees Celsius.

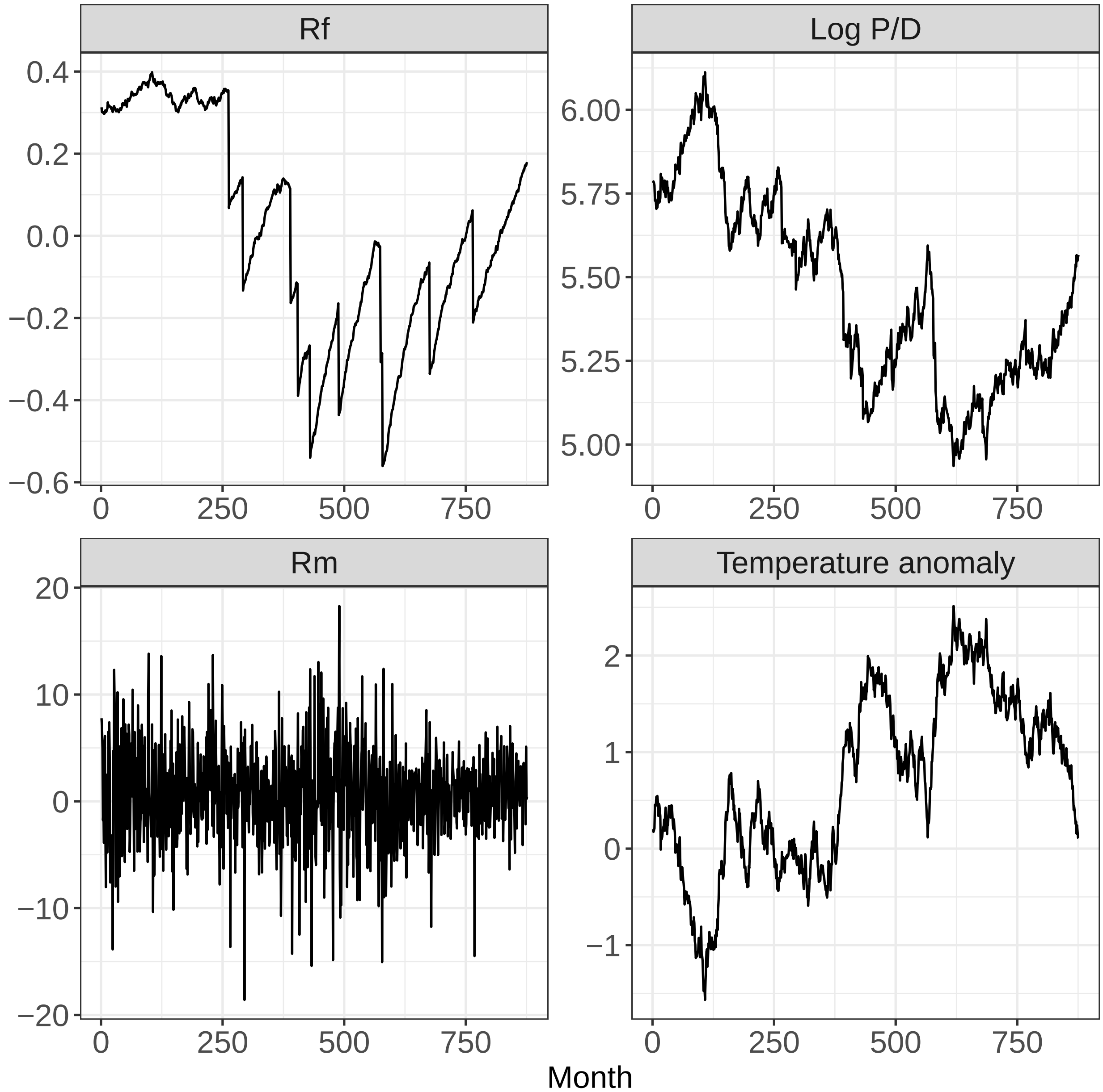

Figure 1: A single simulation from the LRR-T model.
This figure shows the simulated market returns, price-dividend ratios, risk-free returns and temperature anomaly from the calibration of the LRR-T model in Table 1. We simulate 876 months after a burn-in of five years, to match our 1947-2019 historical data sample. Returns are in log percentages, the price-dividend ratio is in logs and the temperature anomaly is in degrees Celsius.

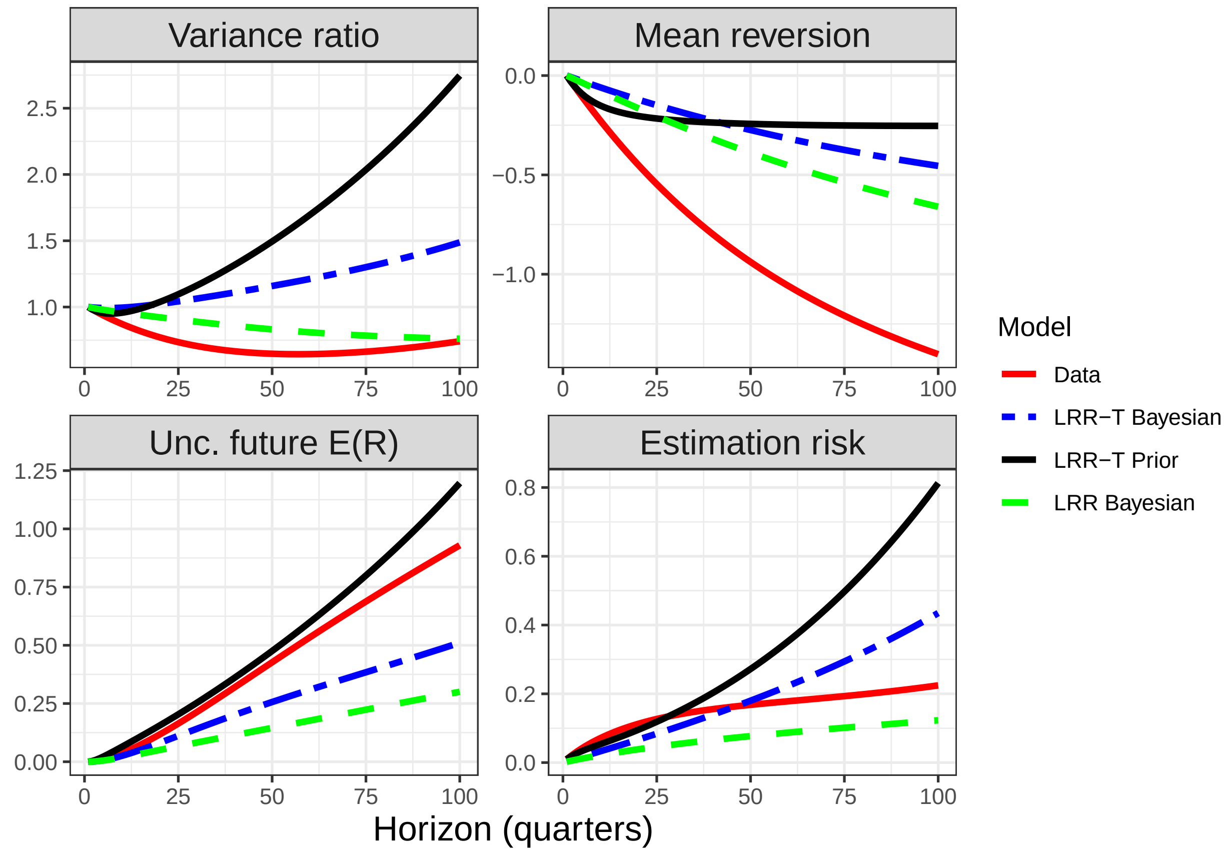

Figure 2: Predictive variance ratios of the market portfolio and its components by horizon. This figure shows the predictive variance ratio of the market portfolio (top left) and the underlying components related to mean reversion (top right), uncertainty about future expected returns (bottom left), and estimation risk (bottom right). Separate variance ratios forecasts for the agnostic (data), dogmatic (LRR-T prior), LRR-T Bayesian, and LRR Bayesian investors are reported based on forecasts from the VARs in panels A to D of Table 3, respectively. These forecasts are made with the Exogenous temperature process from Section 2.2.1. The investment horizons runs from 1 to 100 quarters.

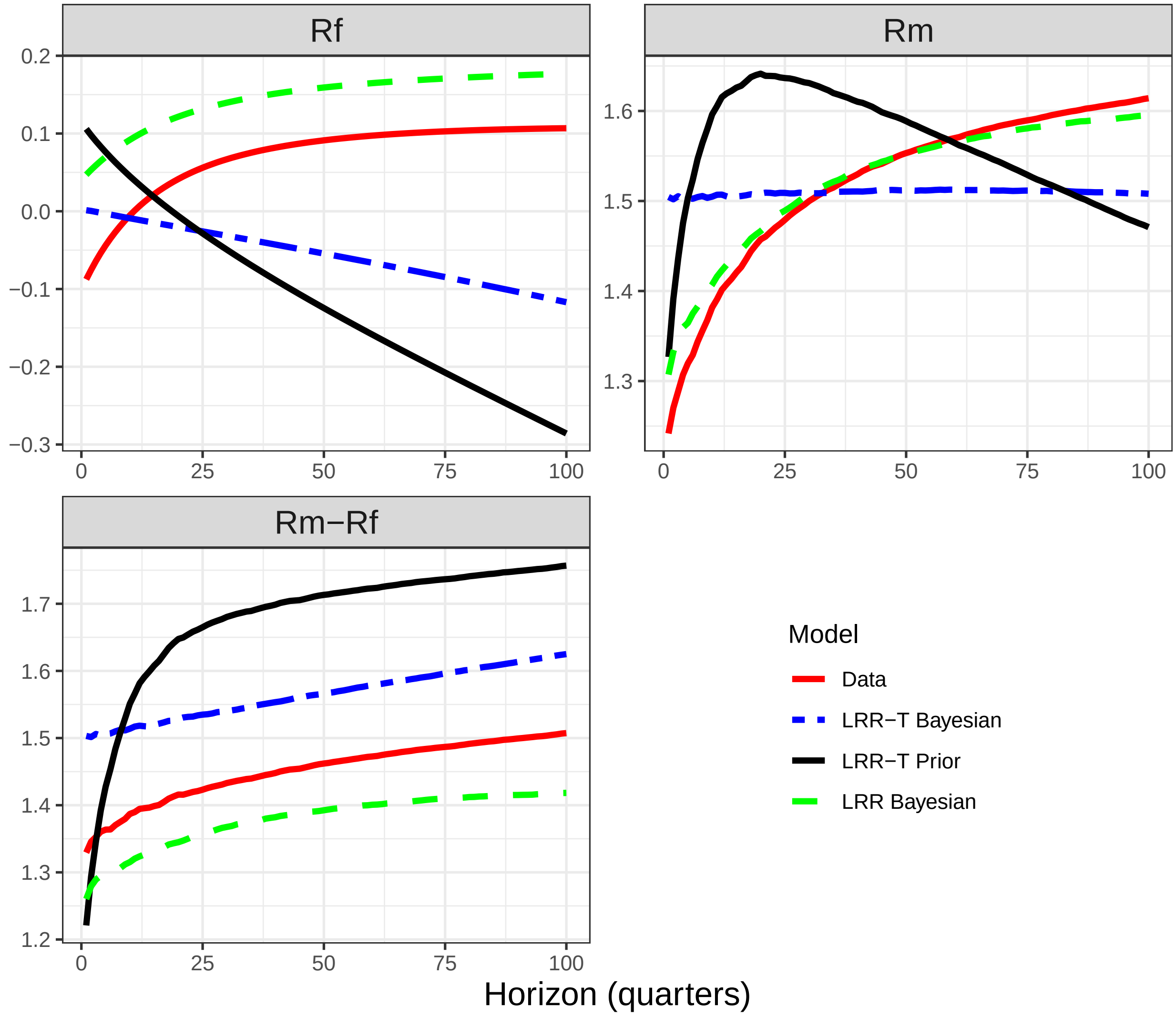

Figure 3: Predictive returns and risk premia.
This figure shows the average forecasted returns on the risk-free asset (top left), the market portfolio (top right), and the related market risk premium (bottom left). Separate return forecasts for the agnostic (data), dogmatic (LRR-T prior), LRR-T Bayesian, and LRR Bayesian investors are reported based on 250,000 forecasts from the VARs in panels A to D of Table 3, respectively. These forecasts are made with the Exogenous temperature process from Section 2.2.1. The investment horizons runs from 1 to 100 quarters. Returns are in log percentages.

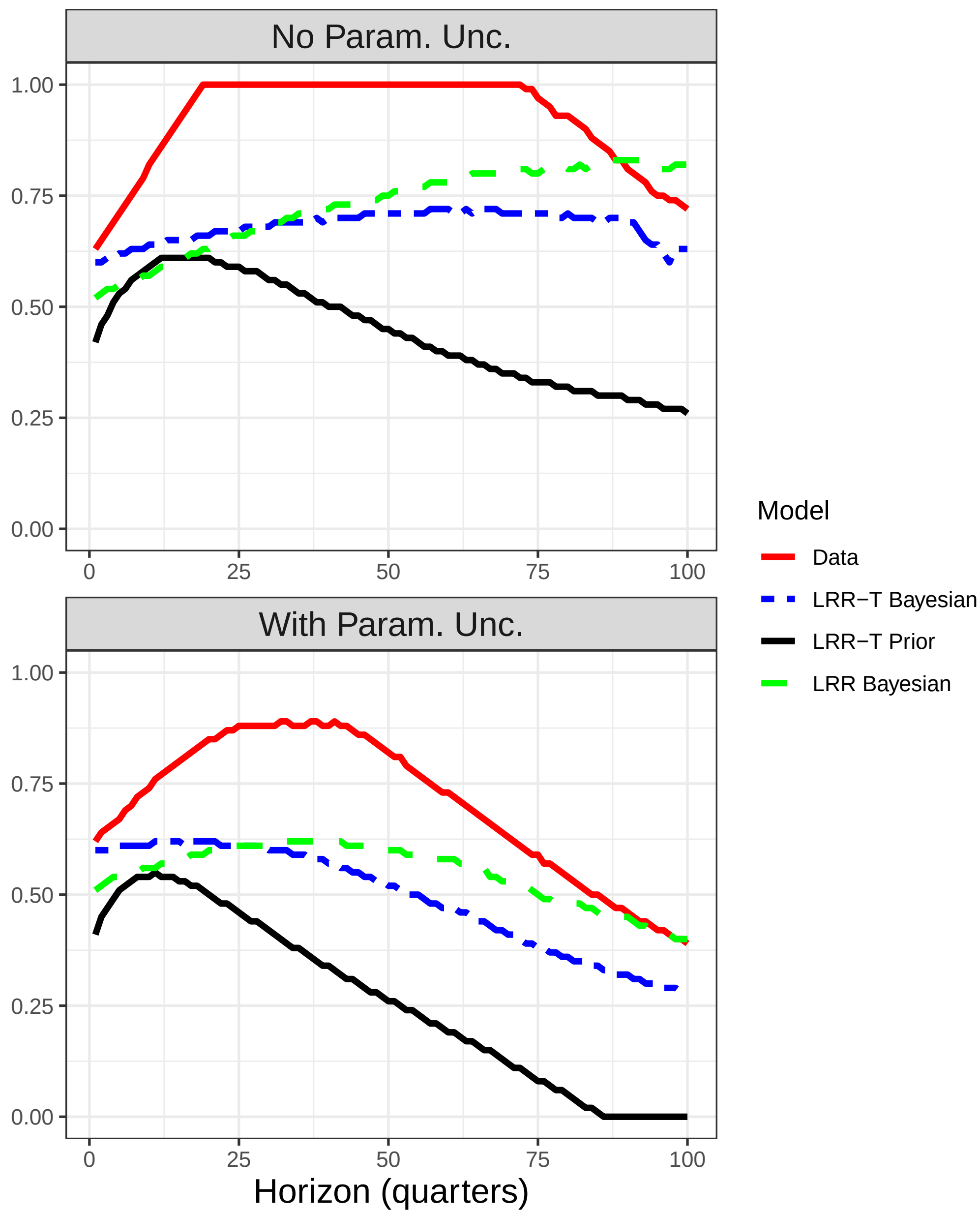

Figure 4: Optimal weight to equities by horizon.
This figure shows the optimal weight to market portfolio for a buy-and-hold investor with risk aversion parameter A=5, based on predictive returns and variances from the VAR models in Table 3 for the agnostic (data), dogmatic (LRR-T prior), LRR-T Bayesian, and LRR Bayesian investors. We optimize the weight to equities with the risk-free asset as alternative investment for investment horizons from 1 to 100 quarters. We do (not) take parameter uncertainty into account in the (top) bottom panel. Short-selling is not allowed.

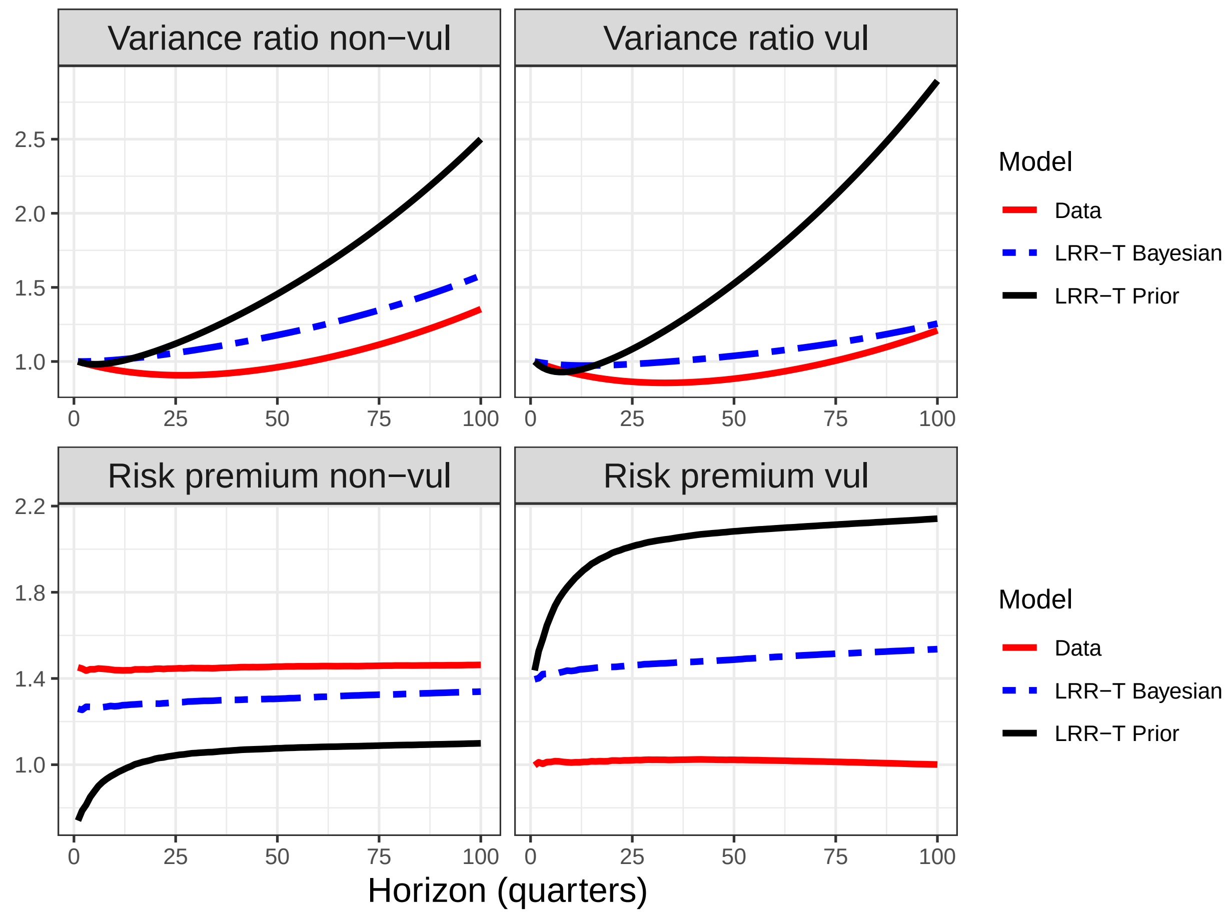

Figure 5: Predictive variance ratios and risk premia for the vulnerable and non-vulnerable portfolios.
This figure shows the predictive variance ratio of the (non-)vulnerable portfolio in the top (left) right panels. The bottom (left) right panels show the risk premia of these portfolios in log percentages per quarter. Separate risk premium and variance ratio forecasts for the agnostic (data), dogmatic (LRR-T prior), and LRR-T Bayesian investors are reported based on forecasts from the regressions in panels A to C of Tables 3 and 5, respectively. These forecasts are made with the Exogenous temperature process from Section 2.2.1. The investment horizons runs from 1 to 100 quarters.

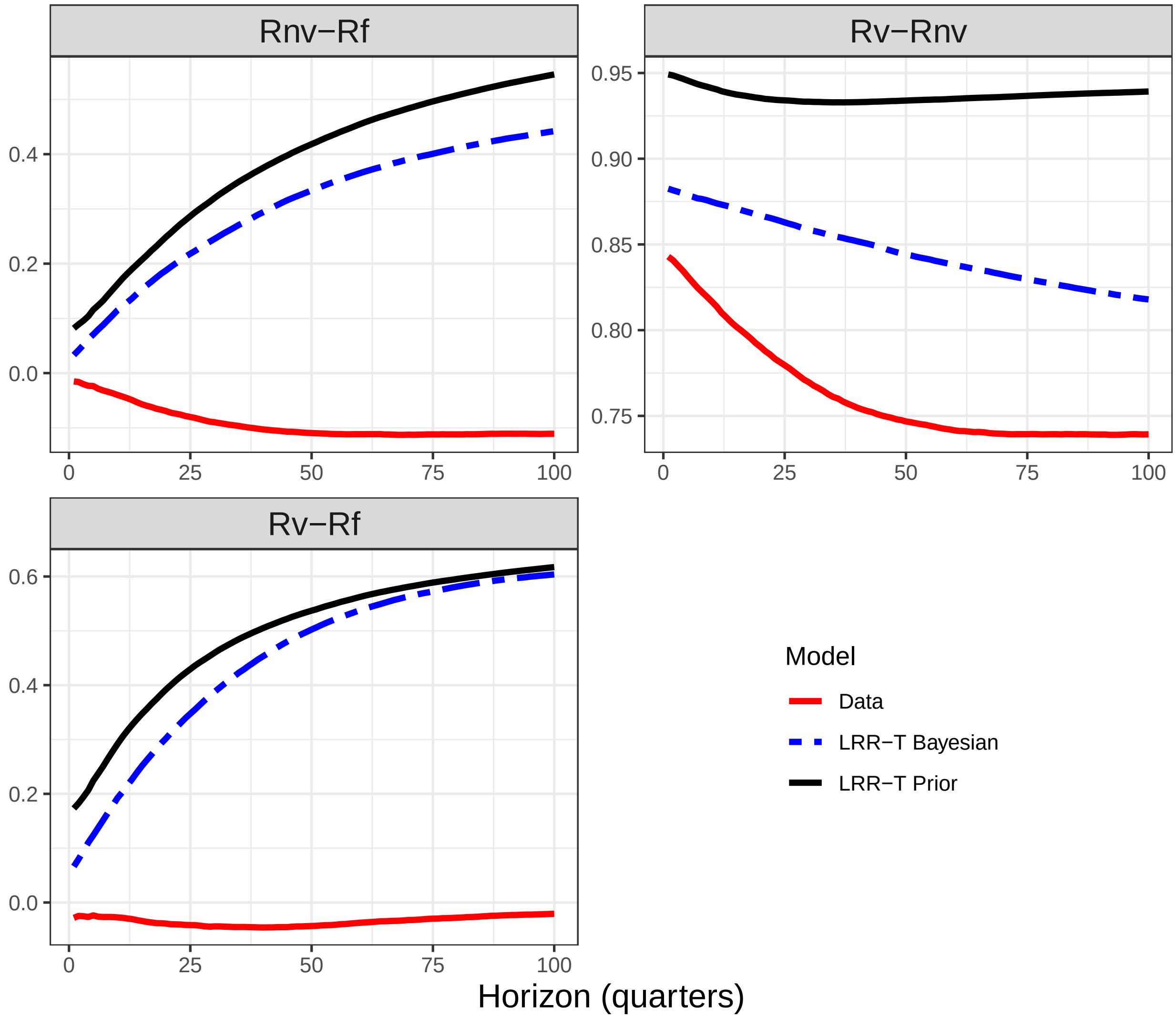

Figure 6: Predictive correlations for returns of the vulnerable and non-vulnerable portfolios. This figure shows the correlations between the 250,000 forecasted cumulative return paths for the vulnerable, non-vulnerable and risk-free portfolios. Separate correlation forecasts for the agnostic (data), dogmatic (LRR-T prior), and LRR-T Bayesian investors are reported based on forecasts from the regressions in panels A to C of Table 3 for the risk-free return and Table 5 for the (non-)vulnerable returns. These forecasts are made with the Exogenous temperature process from Section 2.2.1. The horizons runs from 1 to 100 quarters.

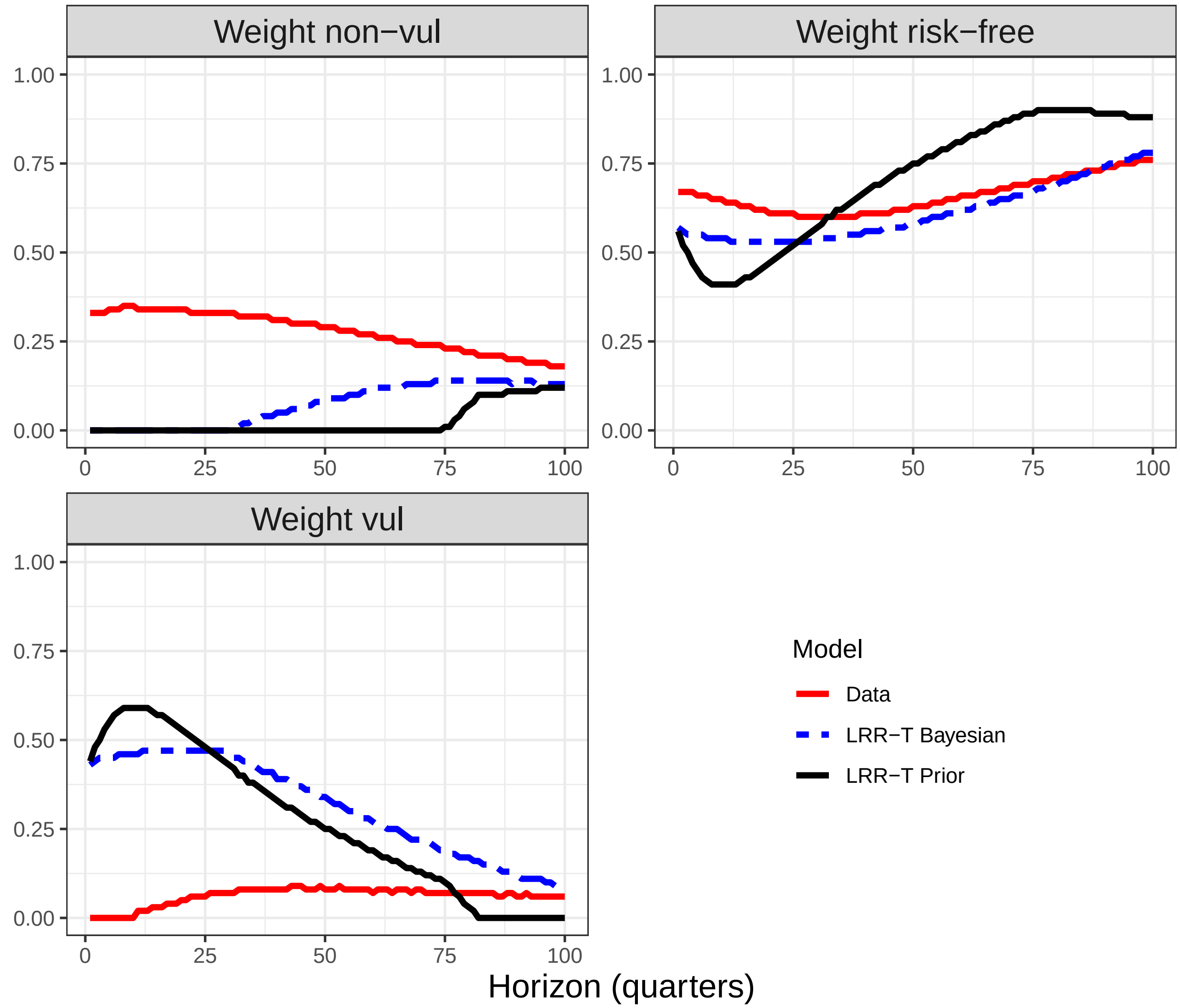

Figure 7: Portfolio choice with vulnerable and non-vulnerable portfolios by horizon. This figure shows the optimal weight to (non-)vulnerable portfolio in the (top) bottom left panel and to the risk-free asset in the top right panel. Optimal portfolios are determined for a buy-and-hold investor with risk aversion parameter A=5, based on predictive returns and variances from the regression models in Tables 3 and 5 for the agnostic (data), dogmatic (LRR-T prior), and LRR-T Bayesian investors. The investor allocates capital between the vulnerable and non-vulnerable portfolios and the risk-free asset for investment horizons from 1 to 100 quarters. Parameter uncertainty is taken into account and short-selling is not allowed.

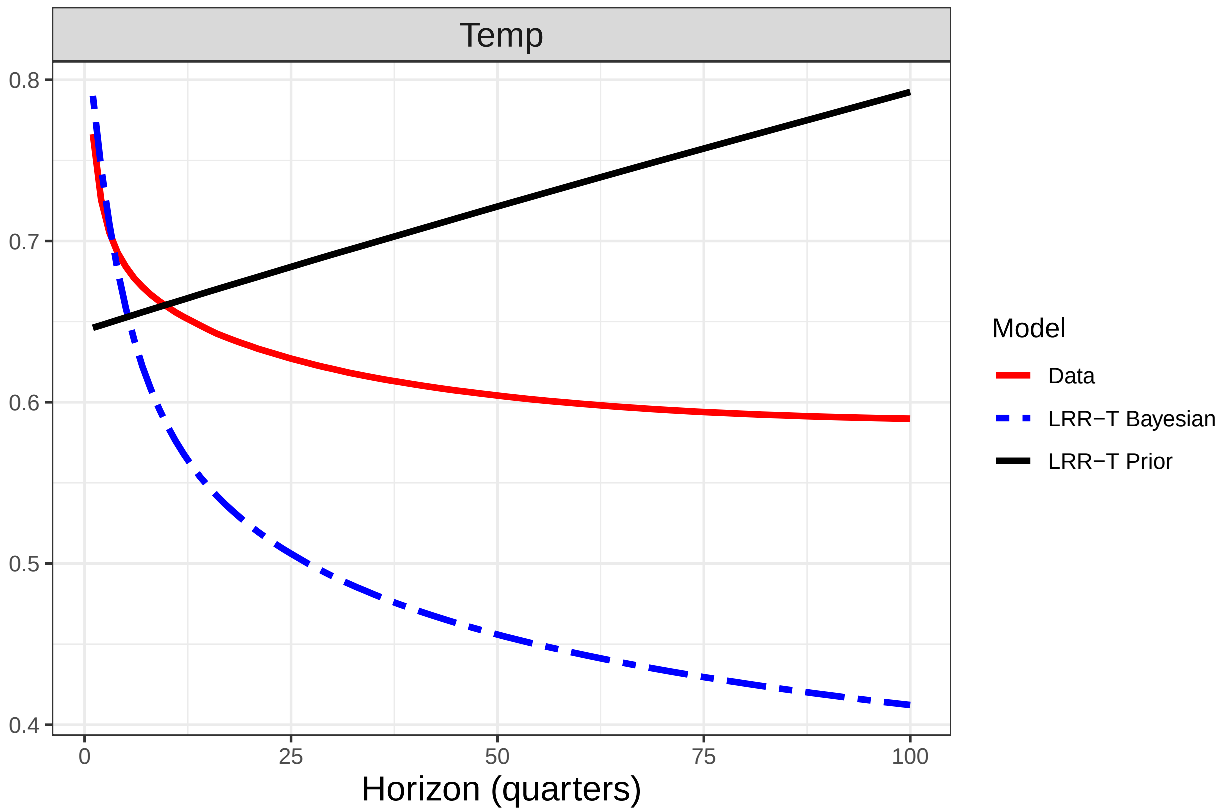

Figure 8: VAR forecasted temperature anomalies.
This figure shows the temperature anomaly forecasts from the agnostic (data), dogmatic (LRR-T prior), and LRR-T Bayesian investors based on forecasts from the VARs from Panel A to C of Table 3. The temperature anomaly is in degrees Celsius. The horizons runs from 1 to 100 quarters.

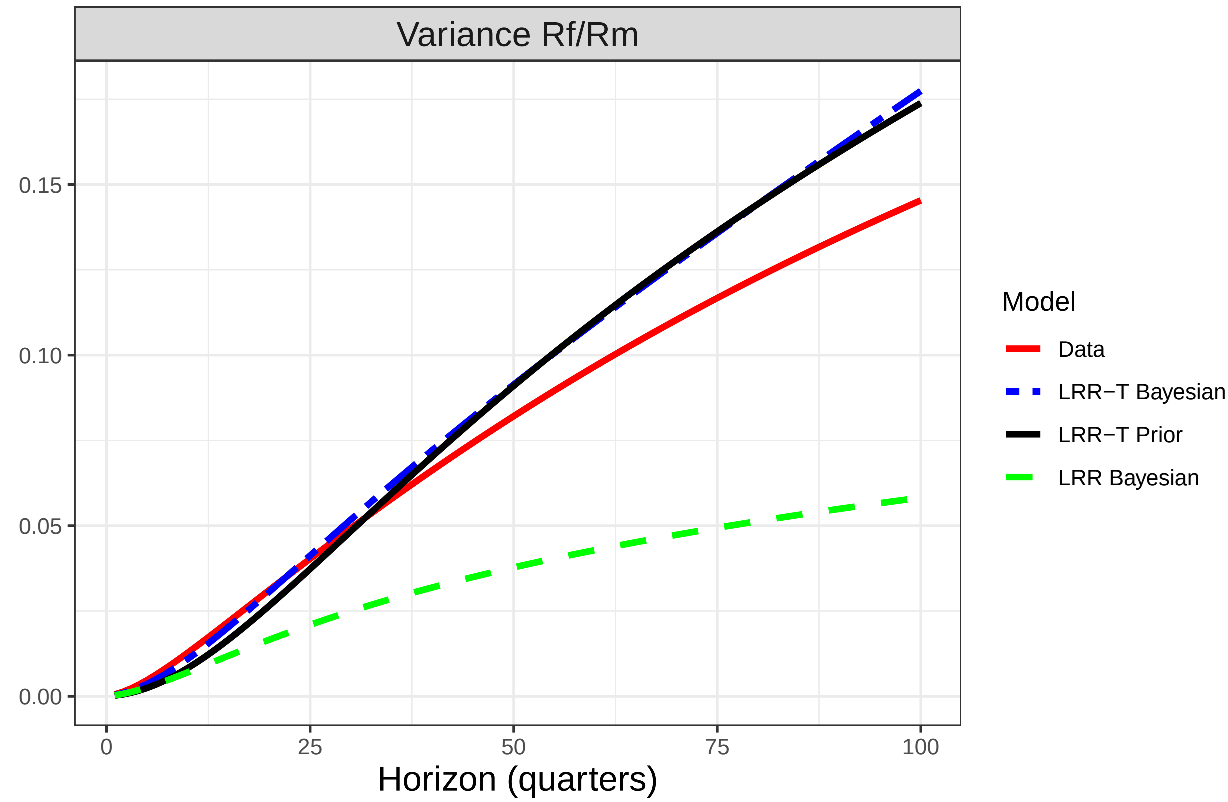

Figure 9: Relative variance of the returns of the risk-free asset and the market portfolio. This figure shows the predictive variance of the returns of the risk-free asset (including roll-over risk) divided by the variance of the returns of the market portfolio. Separate variance forecasts for the agnostic (data), dogmatic (LRR-T prior), LRR-T Bayesian, and LRR Bayesian investors are reported based on forecasts from the predictive regressions in panels A to D of Table 3, respectively. These forecasts are made with the Exogenous temperature process from Section 2.2.1. The investment horizons runs from 1 to 100 quarters.

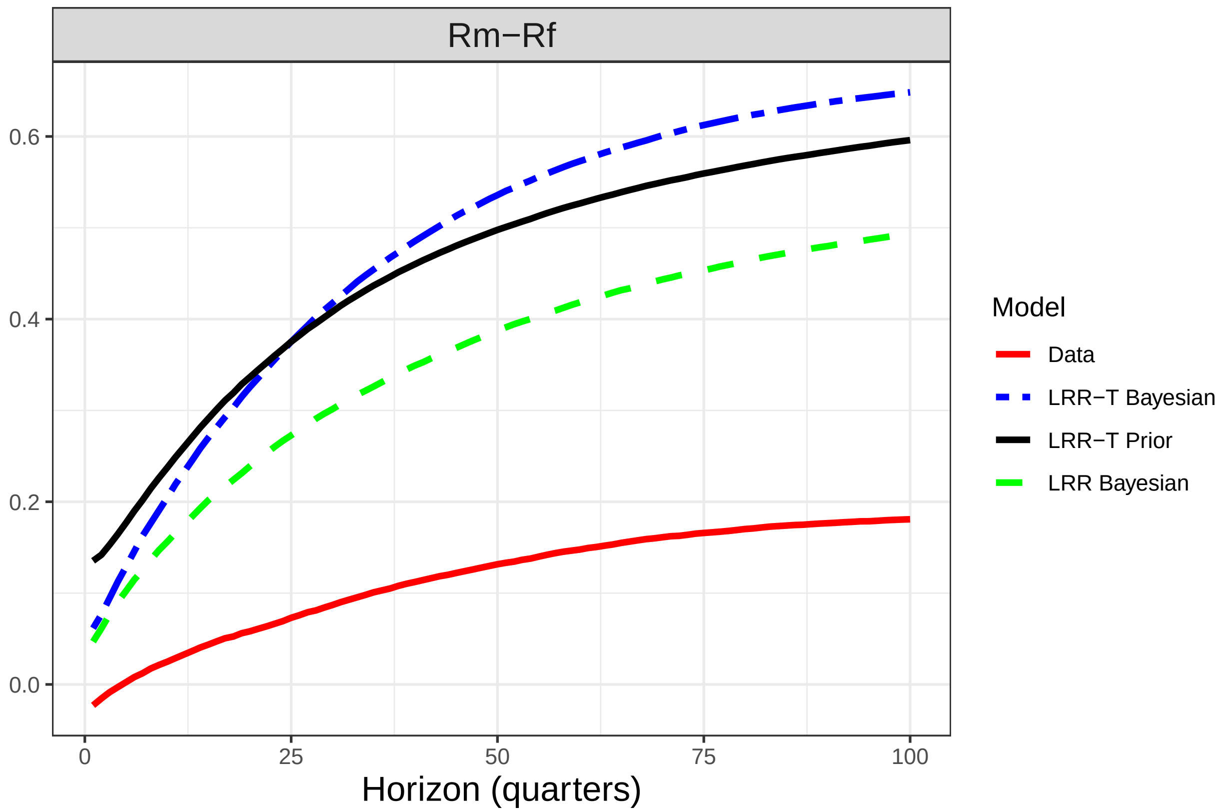

Figure 10: Predictive correlations for returns of the risk-free asset and the market portfolio. This figure shows the correlations between the 250,000 forecasted cumulative return paths for the risk-free asset and the market portfolio. Separate correlation forecasts for the agnostic (data), dogmatic (LRR-T prior), LRR-T Bayesian, and LRR Bayesian investors are reported based on forecasts from the regressions in panels A to D of Table 3. These forecasts are made with the Exogenous temperature process from Section 2.2.1. The horizons runs from 1 to 100 quarters.

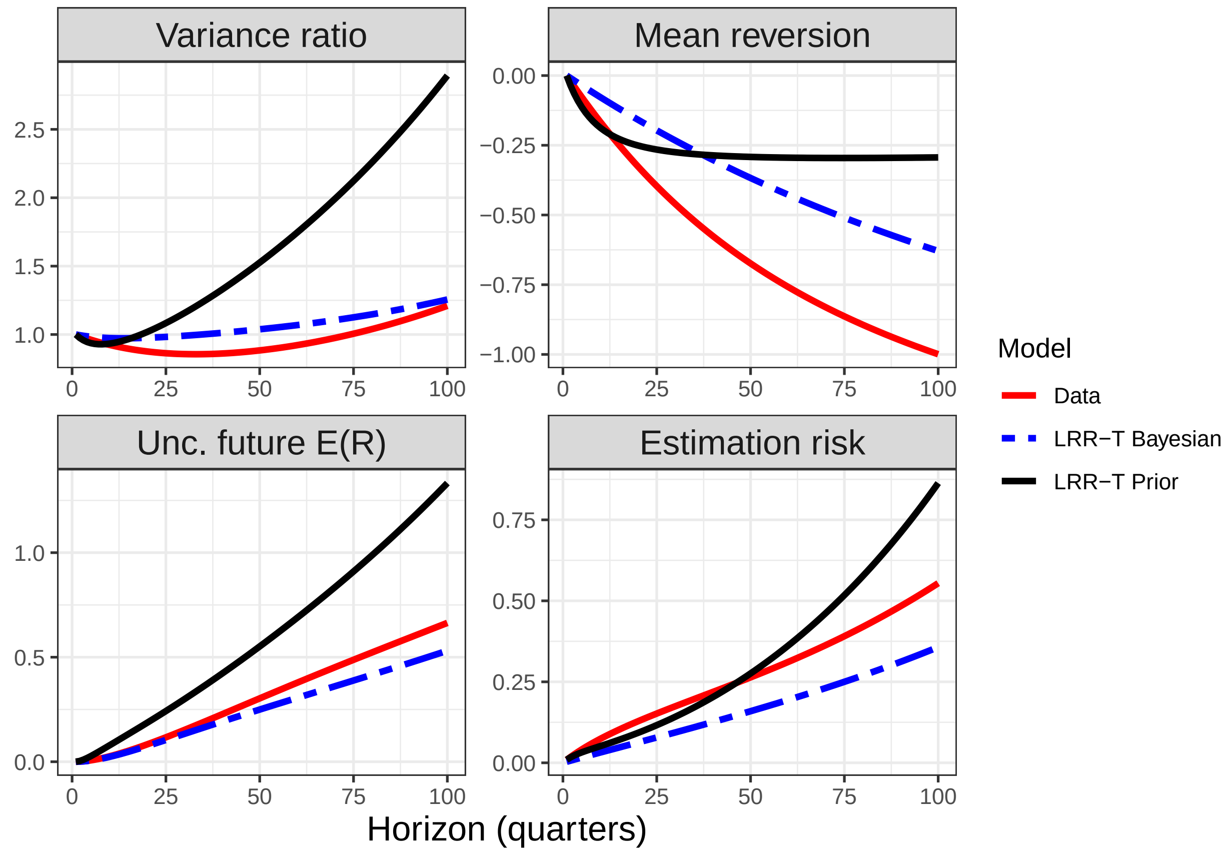

Figure 11: Predictive variance ratios of the vulnerable portfolio.
This figure shows the predictive variance ratio of the vulnerable portfolio (top left) and the underlying components related to mean reversion (top right), uncertainty about future expected returns (bottom left), and estimation risk (bottom right). Separate variance ratios forecasts for the agnostic (data), dogmatic (LRR-T prior), and LRR-T Bayesian investors are reported based on forecasts from the predictive regressions in panels A to C of Table 5, respectively. These forecasts are made with the Exogenous temperature process from Section 2.2.1. The investment horizons runs from 1 to 100 quarters.

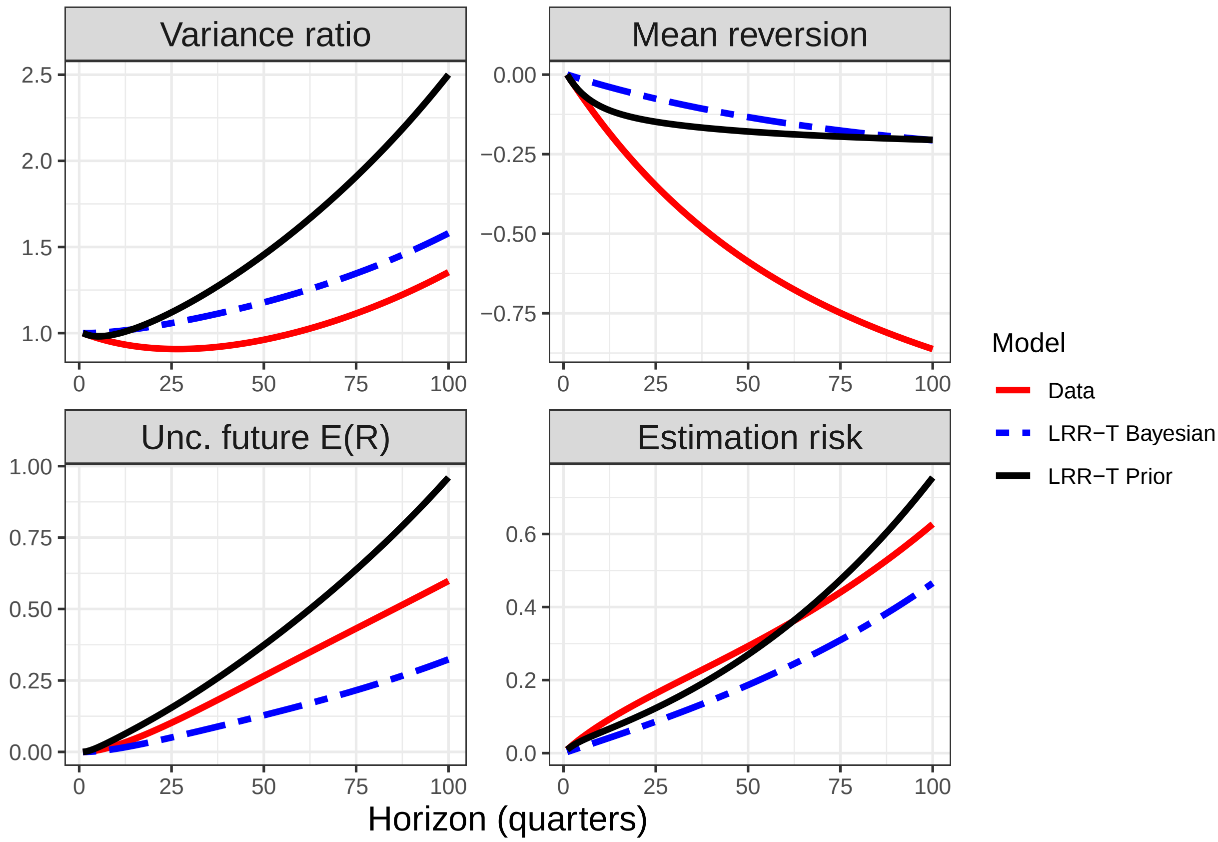

Figure 12: Predictive variance ratios of the non-vulnerable portfolio.
This figure shows the predictive variance ratio of the non-vulnerable portfolio (top left) and the underlying components related to mean reversion (top right), uncertainty about future expected returns (bottom left), and estimation risk (bottom right). Separate variance ratios forecasts for the agnostic (data), dogmatic (LRR-T prior), and LRR-T Bayesian investors are reported based on forecasts from the predictive regressions in panels A to C of Table 5, respectively. These forecasts are made with the Exogenous temperature process from Section 2.2.1. The investment horizons runs from 1 to 100 quarters.

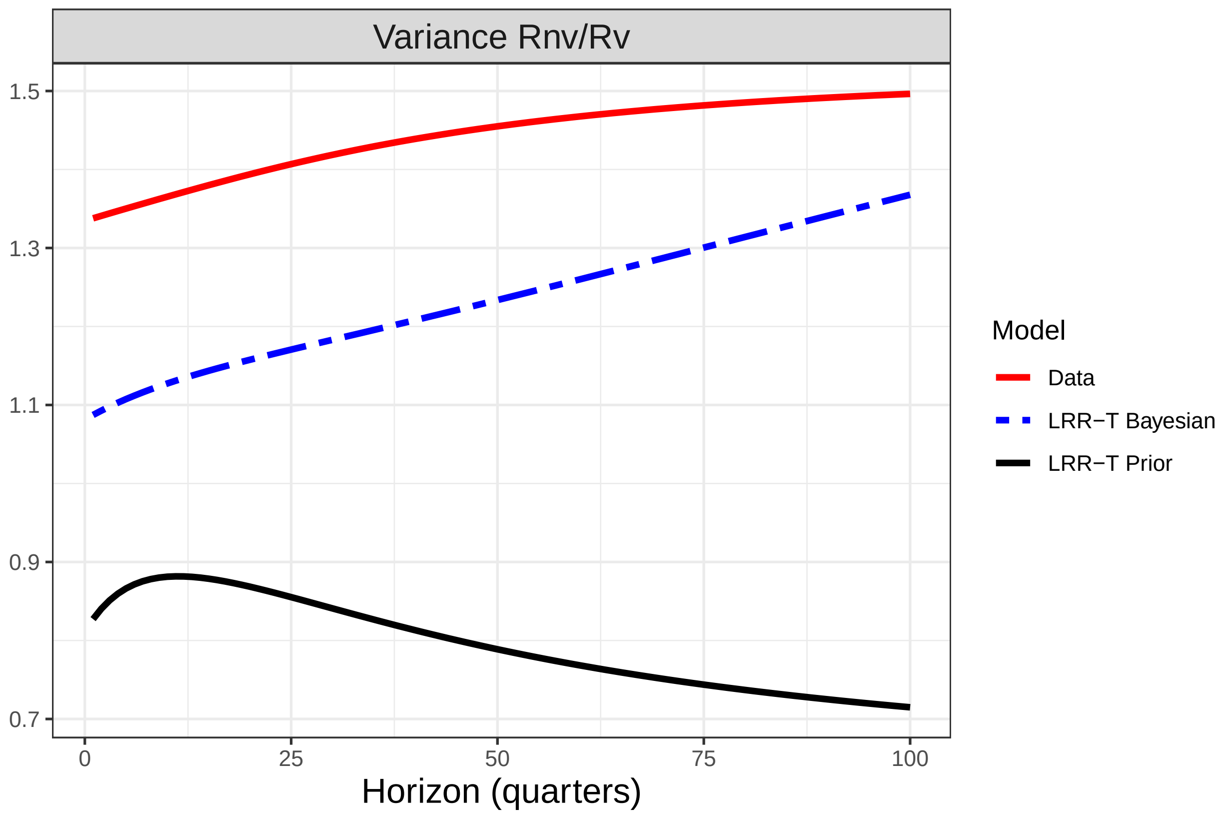

Figure 13: Relative variance of the returns of the non-vulnerable and vulnerable portfolios. This figure shows the predictive variance of the returns of the non-vulnerable portfolio divided by the variance of the returns of the vulnerable portfolio. Separate variance forecasts for the agnostic (data), dogmatic (LRR-T prior), and LRR-T Bayesian investors are reported based on forecasts from the predictive regressions in panels A to C of Table 5, respectively. These forecasts are made with the Exogenous temperature process from Section 2.2.1. The investment horizons runs from 1 to 100 quarters.