

DLC – arbitrage example

DLC: part A & part B, theoretical price ratio 1:1

Day 0: $P(A) = 800$, $P(B) = 1000$

Arb establishes a position by:

- buying 1.25 share of A (\$500 with own capital and \$500 borrowed from broker);
- borrowing 1 share of B and selling the share in the market for \$1000 (short sales proceeds)
- put up \$500 ($=50\% \cdot 1000$) of own capital to fulfill initial margin requirements on short position

The arb incurs transaction costs amounting to 25 basis points (fixed) + 0.5 times the average spread size of 50 basis points is 50 basis points => include in the first return

Assumption: we use minimum margin requirements of 25% for long positions and 30% for short positions as used by Nasdaq and NYSE \$550 ($=25\% \cdot 1.25 \cdot 800 + 30\% \cdot 1000$)

<i>assets</i>		<i>liabilities</i>	
1.25 share of A	\$1000	equity	\$1000
short sales proceeds	\$1000	1 share of B	\$1000
cash margin	\$ 300	margin loan	\$ 500
cash balance	\$ 200		
Total	\$2500	Total	\$2500

Cash balance is the difference between the total amount of collateral and the minimum margin required. The total initial margin is \$2000, consisting of \$1000 short proceeds, \$500 cash margin on the short position and the \$500 “equity margin” on the long position.

Calculation of equity value according to Mitchell, Pulvino & Stafford (2002, *Journal of Finance*, p.560 bottom):

Assets = MV of long (\$1000) + short sales proceeds (\$1000) + cash (\$500) = \$2500

Liabilities = MV of short (\$1000) + margin loan (\$500)

Equity = Assets – Liabilities = \$1000

Return is determined by following assumptions:

- arb receives risk-free rate (T-bill) on cash balance (we assume 5%);
- arb receives 3% per year on the short sales proceeds (this is called the short rebate; our assumptions imply a borrowing fee of 200 basis points, which is considerably larger than the average lending fee estimates provided by Saffi and Sigurdsson (2007, London Business School working paper) for the countries in our sample);
- arb pays risk-free rate (T-bill) plus 50 basis points on margin loan (we assume 5.5%).

Day 1: $P(A) = 900$, $P(B) = 1200$

excess equity on long position = $\$62.5 = (1.25 \cdot 900 - 500 \text{ total equity} - 50\% \cdot 1.25 \cdot 900)$
 [Note: if $P(A)$ would be below the original $P(A)$, excess equity is equal to 0 and if $P(A)$ is smaller than $2/3$ times the original $P(A)$, we would need to use 25%]
 excess margin on short position = $-\$60 = \$1000 \text{ short proceeds} + \$500 \text{ cash on short position} - 130\% \cdot 1200$

Therefore, the cash balance amounts to $\$62.5 - \$60 = \$2.5$

<i>assets</i>		<i>liabilities</i>	
1.25 share of A	\$1125	equity	\$ 925
short sales proceeds	\$1000	1 share of B	\$1200
cash margin	\$497.5	margin loan	\$ 500
cash balance	\$ 2.5		
Total	\$2625	Total	\$2625

Dollar return day 1 to day 2:

- loss of \$75 in equity value
- gain of $1/260$ of 5% on cash balance of \$200;
- gain of $1/260$ of 3% on short sales proceed of \$1000;
- loss of $1/260$ of 5.5% on margin loan of \$500.
- loss of transaction costs amounting to 5 dollar cents per share

Percentage returns from day 0 to day 1 are calculated relative to the equity value on day 0 of \$1000, so:

$$\text{return} = \ln(1000 + (75 + 1/260 \cdot 0.05 \cdot 200 + 1/260 \cdot 0.03 \cdot 1000 - 1/260 \cdot 0.055 \cdot 500 - 0.50\% \cdot (1.25 + 1))) - \ln(1000)$$

Day 2: $P(A) = 1200$, $P(B) = 1800$

excess equity on long position = $\$250 = (1.25 \cdot 1200 - 500 \text{ total equity} - 50\% \cdot 1.25 \cdot 1200)$
 [Note: if $P(A)$ would be below the original $P(A)$, excess equity is equal to 0 and if $P(A)$ is smaller than $2/3$ times the original $P(A)$, we would have to use 25%]
 excess margin on short position = $-\$840 = \$1000 \text{ short proceeds} + \$500 \text{ cash on short position} - 130\% \cdot 1800$

No new margin will be posted; instead part of the position is liquidated. The remaining part is the maximum position that satisfies the minimum margin requirements. This implies that the arb must solve for x (the fraction of the position maintained in the equation:

$$1000 \text{ short proceeds} + 500 \text{ cash on short position} + x \cdot 1.25 \cdot 1200 - 500 + (1-x) \cdot 1.25 \cdot 1200 - (1-x) \cdot 1800 \text{ "excess equity" on the long position (incorporating the fact that extra money needs to be borrowed)} = 50\% \cdot 1.25 \cdot 1200 \cdot x + 130\% \cdot 1800 \cdot x \Rightarrow$$

$$1500 + x \cdot 1.25 \cdot 1200 - 500 + (1-x) \cdot 1.25 \cdot 1200 - (1-x) \cdot 1800 = 3090 \cdot x$$

$$\text{LHS: actual margin after partial liquidation} = \text{RHS: maintenance margin}$$

$$\Rightarrow 700 = 1290 \cdot x \Rightarrow x \approx 0.542$$

The arb sells $(0.458 \times 1.25) = 0.5725$ shares of A for a total amount of \$687. The arb buys 0.458 share of B and returns it to the lender. This costs \$824.4. Hence, the arb needs a net amount of \$137.4 for the transaction, which he will borrow from the broker.

As a check whether the arb fulfills the margin requirements:

excess equity on long position = $-\$93.5 = (1.25 \times 0.542 \times 1200 - 500 \text{ total equity} - 50\% \times 0.542 \times 1.25 \times 1200)$

[Note: if $P(A)$ would be below the original $P(A)$, excess equity is equal to 0 and if $P(A)$ is smaller than $2/3$ times the original $P(A)$, we would have to use 25%]

excess margin on short position = $+\$231.75 = \$1000 \text{ short proceeds} + \$500 \text{ cash on short position} - 130\% \times 0.542 \times 1800$

Note that the cash balance will become positive after this transaction, as the arbitrageur will not receive a margin call on the long position, but the excess cash on the short position will earn interest.

<i>assets</i>		<i>liabilities</i>	
0.542*1.25 share of A	\$ 813	equity	\$ 700
short sales proceeds	\$1000	0.542 share of B	\$ 975.6
cash margin	\$ 268.25	margin loan	\$ 500
cash balance	\$ 231.75	extra loan	\$ 137.4
Total	\$2313	Total	\$2313

New minimum margin is \$1674.78 $(= 50\% \times 0.542 \times 1.25 \times 1200 + 130\% \times 0.542 \times 1800)$

Actual margin: $\$1675.6 = \$1000 \text{ short proceeds} + \$500 \text{ cash on short position and } \$175.6 (= 1.25 \times 0.542 \times 1200 - 500 - 70.5)$ “excess equity” on the long position.

Dollar return day 2 to day 3:

- loss of \$225 in equity value
- gain of $1/260$ of 5% on cash balance of \$2.5;
- gain of $1/260$ of 3% on short sales proceed of \$1000;
- loss of $1/260$ of 5.5% on margin loan of \$500;
- loss of transaction costs amounting to 50bp on the long and short transaction

Percentage returns from day 1 to day 2 are calculated relative to the equity value on day 1 of \$925, so: $\text{return} = \ln(925 + (-225 + 1/260 \times 0.05 \times 2.5 + 1/260 \times 0.03 \times 1000 - 1/260 \times 0.055 \times 500 - 0.50\% \times (0.5725 + 0.458))) - \ln(925)$