# The same bond at different prices: identifying search frictions and selling pressures *

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**Abstract**

I model how corporate bond prices are affected by search frictions and occasional selling pressures, and test my predictions empirically. A key prediction in my model is that in a distressed market with more sellers than buyers, the midprice - the average of bid and ask price - paid by institutional investors is lower than that of retail investors. Using a structural estimation, the model is able to identify liquidity crises based on the relative prices of institutional and retail investors. I identify two liquidity crises (i.e. high number of forced sellers), namely the downgrade of GM and Ford in 2005 and the current crisis. I also find that search costs for institutional investors increase strongly during the subprime crisis, while this is not the case for retail investors. Finally, the model can explain why the spread between corporate bond yields and Treasury yields is so large, the so-called credit spread puzzle.

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# 1 Introduction

The U.S. corporate bond market is a principal source of financing for U.S. firms. While it is bigger than the U.S. Treasury market measured in amount outstanding, trading volume is more than twenty times lower.[1] To trade a corporate bond an investor has to sequentially contact one or several dealers over the telephone. Except for the most actively traded bonds, dealers do not "make a market" and a price quote is firm for only a short period of time which limits the ability to obtain multiple quotations before committing to a trade.[2] Hence, prices reflect a bargaining process that depends on the outside options of investors and dealers, including the relative number of investors currently looking to sell or buy.

This paper seeks to capture these phenomena in a search model, and, using a structural estimation of the model, to identify empirically periods of liquidity shortage as captured by large numbers of selling investors. A key ingredient in the identification is the distribution of prices of the same bond on a given day for buying vs. selling and retail vs. institutional investors.

I find two periods where a large shock to the number of sellers occurred. The first is from March to May 2005 where there was an increasing number of sellers during the three months according to calibration results. This is in line with Acharya, Schaefer, and Zhang (2008) who find that profit warnings of GM and Ford in March and April 2005 and their subsequent downgrade to junk status in May 2005 caused a large sell-off in their bonds. Interestingly, I find a large number of sellers in GM/Ford bonds and other bonds of similar credit quality (possibly due to a sell-off in junk bonds, because some were being substituted by Ford/GM bonds) while investment grade bonds are unaffected. The second period with a large number of sellers according to the empirical results is the subprime crisis beginning in 2007 and lasting to the end of the sample in September 2008. During the crisis there are two peaks in the estimated forced number of sellers; when Bear Sterns is taken over and when Lehman Brothers default.

In the calibration, the estimated search intensity for an investor is determined by

[1]Principal outstanding volume by the end of 2007 was \$5,825bn in the U.S. corporate bond market and \$4,855bn in the U.S. Treasury market while average daily trading volume in 2007 was \$24.3bn in the U.S. corporate bond market and \$567.1bn in the U.S. Treasury market. Source: Securities and Financial Markets Association (www.sifma.org).

[2]See Bessembinder and Maxwell (2008) for further details about the U.S. corporate bond market.

the size of bid-ask spreads at which this investor transacts. I find that transaction costs of retail investors decrease throughout the sample, while costs of institutional investors increase strongly after the onset of the subprime crisis. Thus, it has become more expensive for institutional investors to trade during the crisis, while costs of retail investors have not been affected.

Finally, I examine the impact of search costs and occasional selling pressures on corporate bond yields. I define the premium due to search costs as the midyield for an average investor minus the midyield for an investor who can instantly find a trading partner. The premium represents average yields in the corporate bond market versus average yields in the ultra-liquid Treasury market. The selling pressure premium, I define as the midyield for an average investor under a liquidity shock close to the average during the estimation period minus the midyield in absence of a shock. The search cost premium is large at short bond maturities and small at long maturities, while the reverse is the case for the selling pressure premium. The premia combined can have a variety of shapes as function of maturity and have a size that can explain the 'credit spread puzzle', i.e. the assertion that yield spreads between corporate bonds and Treasury bonds are larger than what can be explained by the default risk of corporate bonds.

The search model I set up is a variant of the model in Duffie, Gârleanu, and Pedersen (2005) (DGP05). An asset is traded, and investors meet market makers with an intensity that depends on the sophistication of the investor. A sophisticated (institutional) investor has a high search intensity while an unsophisticated (retail) investor has a low intensity. Investors are randomly hit by liquidity shocks, and gains from trade arise when investors needing liquidity sell to investors with no liquidity need through a market maker. Once an investor and market maker meet, they bargain over the terms of trade. A market maker immediately unloads a bond in the inter-dealer market and therefore has no inventory costs. There are two key differences between my model and that of DGP05. First, there are cross-sectional differences in the search intensities of investors. This cross-sectional variation leads to differences in prices of institutional investors vs. retail investors, key to identifying the relative number of investors looking to buy or sell. Second, I assume that the asset has a finite maturity, while the asset is infinitely lived in DGP05. As assets mature, firms stand ready to issue new assets. The finite maturity of the asset allows me to study search costs at different times-to-maturity, and this turns out to have important implications.

If there is a shock in funding needs of investors, such that there are more sellers than buyers, the model predicts that midprices of institutional investors are lower than those of retail investors.[3] Sellers and buyers bargain over prices with marketmakers as intermediaries. With more sellers than buyers, prices are set such that sellers are indifferent between selling or keeping the bond. These prices are lower than before the shock. When a shock occurs, I assume that institutional investors only trade with other institutional investors and likewise for retail investors. When they bargain, buyers are in a strong position and sellers in a weak position, because of the excess number of sellers. Institutional investors have better outside options compared to retail investors, because they can find new counterparties more quickly. The combination of strong buyers and stronger outside options for institutional investors relative to retail investors, leads to a stronger price impact of shocks on institutional bid and ask prices compared to those of retail investors. This result is surprising. Duffie, Gârleanu, and Pedersen (2007) find the opposite result; that the price impact of a liquidity shock is higher for unsophisticated investors. The difference in results is due to the presence of marketmakers. In their model, investors trade with each other directly, so a decrease in the number of buyers due to a liquidity shock, leads to an increase in the expected time to find a trading partner. In my model, a liquidity shock does not increase the expected time before a marketmaker is found, but only a fraction of those encounters lead to a trade because there are more sellers than buyers. Thus, in Duffie, Gârleanu, and Pedersen (2007) the price drops because the bargaining position of sellers is worse *and* the expected time for finding a trading partner increases, while my model stresses the bargaining element since the time it takes to find a potential trading partner is the same.

To illustrate the impact of selling pressure on retail and institutional prices, we look at Figure 1. The left-hand graph shows how the midprice of institutional investors is higher than that of retail investors in a normal market. The middle graph illustrates how midprices of institutional investors are lower than those of retail investors in a market with more sellers than buyers. The right-hand graph shows how midprices of institutional investors are markedly lower than prices of retail investors in a day of crisis.

[3] For an in-depth discussion of funding liquidity, and a model that links market liquidity and trader's funding liquidity, see Brunnermeier and Pedersen (2008).

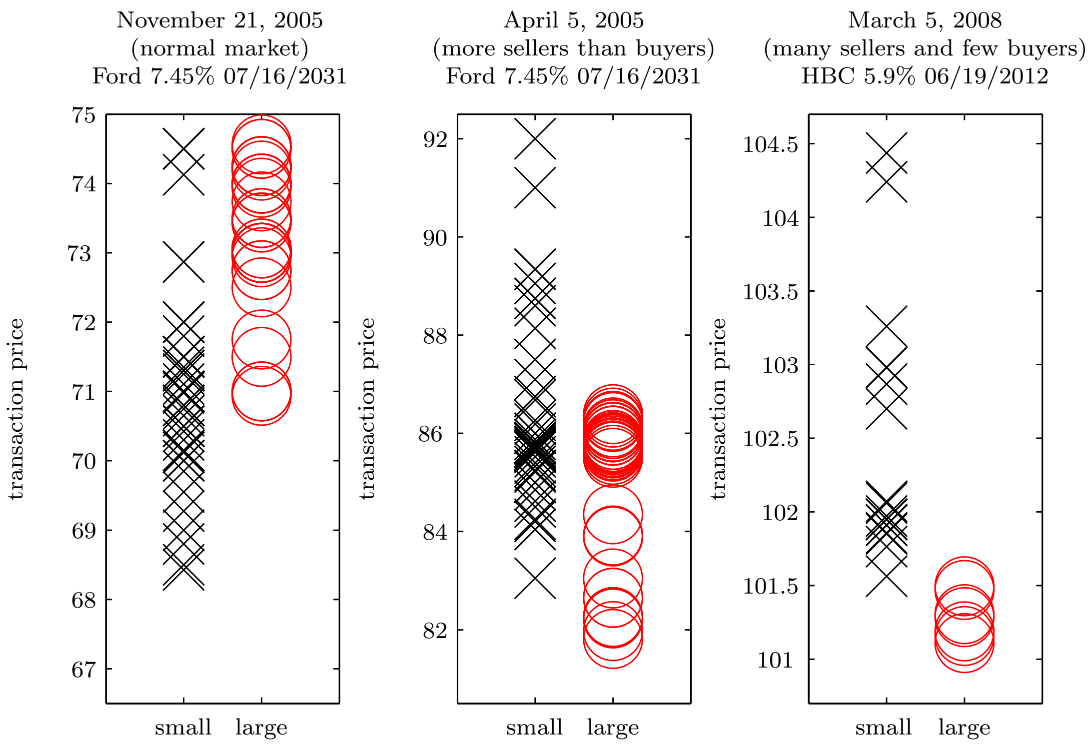


**Figure 1:** *Small and large trades on a normal day, day of distress, and day of crisis.* This graph shows three examples of all trades for a bond during a day smaller than $1,000,000 (marked with crosses) and trades of at least $1,000,000 (marked with circles).

I derive equilibrium prices and estimate by maximum likelihood the parameters of the model using more than 1.5 million corporate bond transactions from the TRACE database for the period October 2004 to September 2008. In the estimation, roundtrip costs are measured using 'unique roundtrip trades' (URT). If two or three trades in a given bond with the same volume happens within 15 minutes and there are no other trades with the same volume on that day, the transactions are part of a URT. The intuition is that either a customer sells to a dealer who sells it to another customer, or a customer sells to a dealer, who sells it to another dealer, who ultimately sells it to a customer. I assume the maximal price in a URT is the ask price and the minimal price the bid price. This measure of transaction costs is most closely related to Green, Hollifield, and Schürhoff (2007)'s 'immediate trades', but their measure requires information about the sell and buy side, which is not available in TRACE. I estimate the search intensity of investors who differ in their degree of sophistication by letting trade size proxy for investor type. Also, I fit the model to demeaned prices. In a given bond on a given day, I define a demeaned bid (ask) price as a transaction bid (ask) price part of a URT minus the average price across prices part of URTs on that day. By fitting to demeaned prices, I focus on the size of bid-ask spreads and the relation between bid and ask prices for different investors.

The outline of the paper is as follows. Section 2 contains the model and my theoretical predictions. Section 3 describes the transaction data and the estimation methodology. Section 4 reports the estimated model and my main empirical findings. Section 5 concludes.

# 2 A search model for the over-the-counter corporate bond market

This section sets up a search model similar to the model in Duffie, Gârleanu, and Pedersen (2005) with two key differences. First, the traded asset has a finite maturity. Since bonds mature and become liquid at maturity, this reflects an important feature of corporate bonds. Second, a range of investors from small retail investors to large institutional investors trade in the corporate bond market, and they trade at different prices. To capture the difference in investor sophistication, there is cross-sectional variation in the search intensities of investors.

I first derive equilibrium bid and ask prices facing investors with different levels of sophistication in finding trading partners. I then define a liquidity shock as a shock increasing the number of sellers relative to the number of buyers and find how prices are affected by such a shock. Finally, I derive how a liquidity shock depress prices faced by sophisticated investors *more* than prices faced by unsophisticated investors.

## 2.1 Model

The economy is populated by two kinds of agents, investors and market makers, who are risk-neutral and infinitely lived. They consume a nonstorable consumption good used as numeraire and their time preferences are given by the discount rate $r > 0$.

Investors have access to a risk-free bank account paying interest rate $r$. The bank account can be viewed as a liquid security that can be traded instantly. To rule out Ponzi schemes, the value $W_t$ of an investor's bank account is bounded from below. In addition, investors have access to an over-the-counter corporate bond market for a credit-risky bond paying coupons at the constant rate of C units of consumption per year. The bond has maturity $T$ and a face value $F$, meaning that it matures randomly with constant intensity $\lambda_T = 1/T$ and pays $F$ at maturity. The bond defaults with intensity $\lambda_D$ and pays a fraction $(1 - f)F$ of face value in default. A bond trade can only occur when an investor finds a market maker in a search process that will be described in a moment.

Investors can hold at most 1 unit of the bond and cannot short-sell. Because agents are risk-neutral, investors hold either 0 or 1 unit of the bond in equilibrium. Investors are heterogeneous in two aspects. First, an investor is of type "high" or "low". The "high" type has no holding cost when owning the asset while the "low" type has a holding costs of $\delta$ per time unit. The holding cost can be interpreted as a funding liquidity shock that has hit the investor. An investor switches from "low" to high" with intensity $\lambda_u$ and from "high" to "low" with intensity $\lambda_d$ and the switching processes are for all investors pair-wise independent. Second, investors differ in the ease with which they find counterparties to trade with. A sophisticated investor quickly finds a trading partner while an unsophisticated investor spends considerable time finding someone to trade with. To capture that different investors trade in different sizes, I assume that bonds sold by an "i"-investor, is bought by an "i"-investor[4].

I assume that there is a unit mass of independent non-atomic market makers who maximize profits. An investor with level of sophistication $i, i \in \{1, 2, ..., I\}$ meets a market maker with intensity $\rho^i$, which can be interpreted as the sum of the intensity of market makers' search for investors and investors' search for market makers. The search intensity is observable to market makers. Without loss a generality I assume that $\rho^i < \rho^j$ when $i < j$, implying that investors with intensity $\rho^1$ are the most unsophisticated and those with intensity $\rho^I$ are the most sophisticated. When an investor and a market maker meet, they bargain over terms of trade. The bargaining will be described in the next section. Market makers immediately unload their positions in an inter-dealer market, so they have no inventory at any time.

The set of investors is $\Gamma = \{ho^i, hn^i, lo^i, ln^i\}_{i=1}^I$ where $h/l$ refers to the "high"/"low" type and $o/n$ to owner/non-owner. There is a continuum of investors and for investors with sophistication level $i$, $\mu_\sigma^i(t)$ denotes the fraction at time $t$ of type $\sigma$. Since the fractions of "$i$"-investors add to 1 at any point in time we have

$$\mu_{ho}^i(t) + \mu_{hn}^i(t) + \mu_{lo}^i(t) + \mu_{ln}^i(t) = 1$$

for every $t$ and any $i = 1, ..., I$. There is also a continuum of credit-risky firms who issue bonds. If a firm defaults it is replaced by an identical new firm. They issue bonds through market makers when there is an excess demand in the market, i.e. when

[4] This assumption is reasonable if dealers do not sell off an initial purchase in smaller blocks. Green, Hollifield, and Schürhoff (2007) report that the majority of trades in the similar municipal bond market are not split. Also, the filtering of trades in the empirical section is likely to filter out most of the splitted trades. The assumption can be relaxed when deriving equilibrium prices, but becomes important when examining prices during liquidity shocks.

$\mu_{hn}^i(t) > \mu_{lo}^i(t)$ for some $i$ and the amount they issue is limited by the speed at which market makers meet investors.

## 2.2 Search equilibrium

The model is solved in two steps. First, asset allocations are determined. This is possible without reference to prices because only low-type owners are sellers and high-type non-owners are buyers, and when a low-type owner or high-type non-owner meets a market maker trade occurs immediately. Bargaining theory tells us that trade occurs immediately [Rubinstein (1982)]. In the following, $i$ defining the level of sophistication is suppressed since the arguments are the same for $i = 1, ..., I$.

The rate of change of mass $\mu_{lo}(t)$ of low-type owners is

$$\dot{\mu}_{lo}(t) = -\rho\mu_m(t) - (\lambda_T + \lambda_D)\mu_{lo}(t) - \lambda_u\mu_{lo}(t) + \lambda_d\mu_{ho}(t) \tag{1}$$

where $\mu_m(t) = \min\{\mu_{hn}(t), \mu_{lo}(t)\}$. Market makers buy from *lo* investors and sell to *hn* investors instantly through the inter-dealer market. If a *lo* investor meets a market maker with intensity $\rho$ and if $\mu_{lo}(t) \leq \mu_{hn}(t)$ all meetings lead to a trade and the *lo* investor becomes a *ln* investor. However, if $\mu_{lo}(t) > \mu_{hn}(t)$ not all meetings result in a trade. The first term $-\rho\mu_m(t)$ in (1) reflects this fact. A *lo* investor becomes a *ln* investor if the owned bond either matures or defaults, and the second term in (1), $-(\lambda_T + \lambda_D)\mu_{lo}(t)$, reflects this. The third term is present because *lo* investors switch type to *ho* with intensity $\lambda_u$, and the last term is due to investors switching from type *ho* to *lo*.

Derivations of the rates of change of mass of the other investor types are very similar. The only slight difference is that although sellers might be rationed, buyers are not. If there are not enough sellers, firms step in and issue more debt. The rates of change are therefore given as

$$\dot{\mu}_{ho}(t) = \rho\mu_{hn}(t) - (\lambda_T + \lambda_D)\mu_{ho}(t) + \lambda_u\mu_{lo}(t) - \lambda_d\mu_{ho}(t) \tag{2}$$

$$\dot{\mu}_{hn}(t) = -\rho\mu_{hn}(t) + (\lambda_T + \lambda_D)\mu_{ho}(t) + \lambda_u\mu_{ln}(t) - \lambda_d\mu_{hn}(t) \tag{3}$$

$$\dot{\mu}_{ln}(t) = \rho\mu_m(t) + (\lambda_T + \lambda_D)\mu_{lo}(t) - \lambda_u\mu_{ln}(t) + \lambda_d\mu_{hn}(t), \tag{4}$$

and in Appendix A an explicit expression for the steady state mass of the four investor types is given.

## 2.3 Equilibrium Prices

Next, we determine the prices that prevail in steady state: a) the bid price $B_t$ at which investors sell to market makers, b) the ask price $A_t$ at which investors buy from market makers, and c) the inter-dealer price. Each investor's utility for future consumption depends only on his current type $\sigma(t) \in \Gamma$ and wealth $W_t$ in his bank account. Lifetime utility is

$$U(W_t, \sigma(t)) = \sup_{\zeta,\theta} E_t \int_t^{\infty} e^{-rs} d\zeta_{t+s} \tag{5}$$

subject to

$$dW_t = rW_t dt - d\zeta_t + \theta_t (C - \delta 1_{\{\sigma^\theta(t)=lo\}})dt - \hat{P}_t d\theta_t \tag{6}$$

where $\zeta$ is a cumulative consumption process, $\theta_t \in \{0,1\}$ is a feasible holding process, $\sigma^\theta$ is the type process induced by $\theta$, and at the time of a possible holding change, $\hat{P}_t \in \{A_t, B_t, F, (1-f)F\}$ is the "trade price". From (5) and (6) we have that lifetime utility is $W(t) + V_{\sigma(t)}$ where

$$V_{\sigma(t)}(t) = \sup_{\theta} E_t[\int_t^{\infty} e^{-r(t-s)} \theta_s (C - \delta 1_{\{\sigma^\theta(s)=lo\}})ds - e^{-r(s-t)} \hat{P}_s d\theta_s].$$

The appendix shows that the value functions satisfy the equations

$$\begin{aligned}
\dot{V}_{ln} &= rV_{ln} - \lambda_u (V_{hn} - V_{ln}) && (7)\\
\dot{V}_{hn} &= rV_{hn} - \lambda_d (V_{ln} - V_{hn}) - \rho(V_{ho} - V_{hn} - A) && (8)\\
\dot{V}_{lo} &= rV_{lo} - \lambda_u (V_{ho} - V_{lo}) - \rho(B + V_{ln} - V_{lo}) - \lambda_T (F + V_{ln} - V_{lo}) \\
&\qquad -\lambda_D ((1-f)F + V_{ln} - V_{lo}) - (1-\delta) && (9)\\
\dot{V}_{ho} &= rV_{ho} - \lambda_d (V_{lo} - V_{ho}) - \lambda_T (F + V_{hn} - V_{ho}) \\
&\qquad -\lambda_D ((1-f)F + V_{hn} - V_{ho}) - 1 && (10)
\end{aligned}$$

A low-owner investor is willing to sell to a market maker if the price is at least $\Delta V_l = V_{lo} - V_{ln}$ while a high-nonowner is willing to buy from a market maker if the price is no more than $\Delta V_h = V_{ho} - V_{hn}$. Likewise, the market maker is willing to buy if the price is no more than the inter-dealer price M (at which he immediately unloads the bond in the inter-dealer market), and willing to sell if the price is no less than M. If sellers are not constrained the bid price $B$ can be any price between seller's reservation value $\Delta V_l$ and the inter-dealer price $M$. In contrast, if sellers are constrained the bid price is seller's reservation value, since sellers must be indifferent to trading or not if some trade and others do not. Buyers are per construction not constrained since firms issue bonds if there is a demand, and the ask price $A$ is between buyer's reservation value $\Delta V_h$ and the inter-dealer price $M$. If sellers are not constrained, Nash bargaining between investor and market maker leads to bid and ask prices

$$A = \Delta V_h z + M(1-z) \tag{11}$$

$$B = \Delta V_l z + M(1-z) \tag{12}$$

where $z$ is the bargaining power of the market maker. Duffie, Gârleanu, and Pedersen (2007) show that a bargaining outcome if this kind can be justified by an explicit bargaining procedure.

If sellers are constrained the bid B and inter-dealer price M equals sellers' reservation price $\Delta V_l$. If they are not constrained, any inter-dealer price between $\Delta V_l$ and $\Delta V_h$ can be an outcome in the model, and I let $M = \tilde{q}\Delta V_h + (1-\tilde{q})\Delta V_l$ where $0 \leq \tilde{q} \leq 1$. If $\tilde{q}$ is close to one, the ask price is close to the reservation value of the buyer and the market can be interpreted as a sellers' market, while a $\tilde{q}$ close to zero can be interpreted as a buyers' market.

A simple example illustrates the effect of $z$ and $\tilde{q}$. Consider a buyer who is willing to buy at a maximal price of $\Delta V_h = 103$, a seller who is willing to sell at a minimal price of $\Delta V_l = 100$, and a market maker who is intermediating the trade. The gains of trade of 103-100=3 is to be split between the three agents. Assume that $z = 0.5$ and $\tilde{q} = 0.75$. The inter-dealer price is 102.25, the bid price is 101.125, and the ask price is 102.625. The dealer gains the bid-ask spread $z * 3 = 1.5$, so $z$ measures the part of the total gain the dealer is receiving. The seller gains 1.125 while the buyer gains 0.375, so $\tilde{q}$ determines how the rest of the gain is split between buyer and seller. Consider now $\tilde{q} = 0.25$. In this case the bid price is 100.375 and the ask price 101.875. The bid-ask spread is the same as before, but bid and ask prices are lower.

The following theorem states the equilibrium bid and ask prices in the economy, and a proof is given in the Appendix.

**Theorem 2.1. (Prices in equilibrium).** *Assume that all investors face the same inter-dealer price and trade in equilibrium. The bid $B^i$ and ask $A^i$ prices for investor $i$*

*with search intensity $\rho^i$ are given as*

$$
\begin{aligned}
A^i &= \Delta V_h^i z + M(1-z) \\
B^i &= \Delta V_l^i z + M(1-z)
\end{aligned}
$$

*where*

$$
\begin{aligned}
\Delta V_l^i &= \frac{\rho^i(1-z)M + \lambda_T F + \lambda_D(1-f)F + C}{r + (1-z)\rho^i + \lambda_T + \lambda_D} \\
&\quad - \frac{\delta(r + \lambda_T + \lambda_D + \lambda_d + (1-z)\rho^i)}{(r + \lambda_T + \lambda_D + \rho^i(1-z))(\lambda_u + \lambda_d + \rho^i(1-z) + r + \lambda_T + \lambda_D)} \\
\Delta V_h^i &= \Delta V_l^i + \frac{\delta}{r + (1-z)\rho^i + \lambda_T + \lambda_D + \lambda_d + \lambda_u} \\
M &= \frac{C + \lambda_T F + \lambda_D(1-f)F}{r + \lambda_T + \lambda_D} - \frac{\delta(\lambda_d + (1-\tilde{q})[r + (1-z)\rho_0 + \lambda_T + \lambda_D])}{(r + \lambda_T + \lambda_D)(r + (1-z)\rho_0 + \lambda_d + \lambda_u + \lambda_T + \lambda_D)}
\end{aligned}
$$

*and* $\rho_0 = \min(\rho_1, ..., \rho_I)$ *and* $0 \leq \tilde{q} \leq 1$.

I assume in the theorem that the interdealer price is the same for all bonds. Alternatively, the assumption that bonds can only be traded (through a marketmaker) between "i"-investors can be relaxed, and in this case it follows automatically that there is only one interdealer price and the results in the theorem holds as well. As an obvious but important consequence of the theorem we have the following corollary.

**Corollary 2.1. (Bid-ask spreads).** *The bid-ask spread for investor i with search intensity $\rho_i$ is given as*

$$
A^i - B^i = \frac{z\delta}{\lambda_u + \lambda_d + \rho^i(1-z) + r + \lambda_T + \lambda_D} \tag{13}
$$

There are several interesting implications regarding bid-ask spreads. First, bid-ask spreads decrease with the level of investor sophistication $\rho$. Sophisticated investors trade at tight bid-ask spreads while unsophisticated investors trade at wide bid-ask spreads. Prices are determined through a bargaining process. The threat of cutting off negotiations and finding another trading partner is stronger for sophisticated investors (they find a new trading partner more easily) and therefore they negotiate tighter bid-ask spreads. This insight is also discussed in Duffie, Gârleanu, and Pedersen (2005). Second, bid-ask spreads for a given issuer decrease in the maturity of the bond $1/\lambda_T$. When a bond matures, it becomes liquid, and this influences the bargaining power of investors. A seller is almost indifferent between selling a short-maturity bond or not, since the bond will soon be converted to cash. A buyer is almost indifferent between buying or not, since the buyer will have to search for a new bond to buy when the bond matures. Therefore, as maturity goes to zero, so does the bid-ask spread. Third, the bid-ask spread is decreasing in the default probability of the issuer. The reason for this is the same as that of maturity; as default intensity increases, the firm is more likely to default and a fraction of the notional of the bond is repayed. Thus, the bond becomes "liquid" at default and is similar to a bond with short maturity. This effect is likely to be very small except for very credit-risky bonds.

Next, I define a liquidity shock to investors. I assume that the fractions of investors are in steady state and a sudden liquidity shock occurs. If a shock of size $0 \leq s \leq 1$ occurs a "high"-investor (no liquidity need) becomes a "low"-investor (liquidity need) with probability $s$:

**Definition 2.1. (Liquidity shock)**. *Assume that the fractions of types of investors with search intensity $\rho$ are in steady state, denoted $\mu_{ho}^{ss}, \mu_{hn}^{ss}, \mu_{lo}^{ss}$, and $\mu_{ln}^{ss}$. When a liquidity shock of size $0 < s \leq 1$ occurs, any high-investor becomes a low-investor with probability $s$. The fractions of types immediately after the shock are $\mu_{ho}(s) = (1-s)\mu_{ho}^{ss}$, $\mu_{hn}(s) = (1-s)\mu_{hn}^{ss}$, $\mu_{lo}(s) = \mu_{lo}^{ss} + s\mu_{ho}$, and $\mu_{ln}(s) = \mu_{ln}^{ss} + s\mu_{hn}$.*

Prices following a liquidity shock are given in the following theorem and a proof is in the Appendix.

**Theorem 2.2. (Prices after a liquidity shock)**. *Assume that a liquidity shock of size $0 < s \leq 1$ occurs to all investors type and that $\rho^i + \lambda_T + \lambda_D > \lambda_d + \lambda_u$. If $s \leq \frac{\lambda_T+\lambda_D}{\rho^i+\lambda_T+\lambda_D}$ bid and ask prices do not change after the liquidity shock. If $s > \frac{\lambda_T+\lambda_D}{\rho^i+\lambda_T+\lambda_D}$ bid and ask prices immediately after the shock are*

$$\begin{aligned} B^i(s) &= e^{-t_s^i(r+\lambda_D+\lambda_T)}\Delta V_l^i + (1 - e^{-t_s^i(r+\lambda_D+\lambda_T)})\Delta V_l^{i,imb} \\ A^i(s) &= B + \frac{z\delta}{\lambda_u + \lambda_d + \rho^i(1-z) + r + \lambda_T + \lambda_D} \end{aligned}$$

*where $\Delta V_l^i$ is given in Theorem 2.1,*

$$\Delta V_l^{i,imb} = \frac{C + \lambda_T F + \lambda_D(1-f)F}{r + \lambda_T + \lambda_D} - \frac{\delta(r + \lambda_T + \lambda_D + \lambda_d + (1-z)\rho^i)}{(r + \lambda_T + \lambda_D)(\lambda_u + \lambda_d + \rho^i(1-z) + r + \lambda_T + \lambda_D)},$$

*and $t_s^i$ is the unique solution to*

$$0 = 1 - se^{-(\lambda_u+\lambda_d)t_s^i} - \frac{\rho^i}{\rho^i + \lambda_T + \lambda_D}e^{-(\lambda_T+\lambda_D)t_s^i}.$$

The reservation price $\Delta V_l^{i,imb}$ in the theorem is the reservation price of an "i"-investor in a situation where there are more sellers than buyers at all points in the future. Thus, it is the price a seller would be willing to sell at, if she knew that she would be constantly constrained. The theorem shows that seller's reservation price is a linear combination of this reservation price and that in equilibrium where sellers are not constrained. The weight on $\Delta V_l^{i,imb}$ depends on the amount of time that sellers are constrained after the shock. The larger the shock $s$ is, the longer the period of constrained sellers is, and the lower the prices following the shock are.

Sophisticated investors are hit faster by a liquidity shock than unsophisticated investors. Due to search frictions, the allocation of bonds is more inefficient for the unsophisticated investors, meaning that a larger fraction of "high"-investors do not own bonds. A liquidity shock therefore leads to a smaller order imbalance for unsophisticated investors since a smaller fraction of the shocked investors wishes to sell bonds.

From the theorem we also see that bid-ask spreads after the shock is the same as those in equilibrium, which we state in the theorem below. Thus, while prices change following a shock, bid-ask spreads do not. The decrease in prices facing unsophisticated and sophisticated investors is not the same. As the next theorem shows, the midprice (i.e. average of bid and ask price) facing sophisticated investors decreases *more* than the midprice facing unsophisticated investors.

**Theorem 2.3. (Relation between prices after a liquidity shock)**. *If a liquidity shock of size $0 < s \leq 1$ occurs to any two investors of type $i$ and $j$ where $\rho_i < \rho_j$, the following holds:*

1. *Bid-ask spreads of investors are not affected by the liquidity shock.*
2. *Assume that $\lambda_u > \frac{1-z}{z}\lambda_d$ and $\rho_i$ and $\rho_j$ are sufficiently high*[5]*. For any shock size $\frac{\lambda_T+\lambda_D}{min\{\rho_i,\rho_j\}+\lambda_T+\lambda_D} < s \leq 1$ midprices $M(s) = \frac{1}{2}(A(s) + B(s))$ at which investors trade satisfy that $M_i(s) - M_j(s)$ is increasing in $s$.*

The theorem shows that for any shock above a minimum threshold, the difference between the midprice of unsophisticated investors and the midprice of sophisticated investors is a monotonously increasing function of the shock size. This threshold is for most cases small. To illustrate the effect of liquidity shocks on prices, the top graph in Figure 2 shows bid and ask prices for a 10-year bond for a sophisticated and unsophisticated investor as a function of size of liquidity shock. We see that prices

[5] The precise condition for $\rho_i$ and $\rho_j$ being "sufficiently high" is that $(1-z)\rho\lambda_u(\rho+\lambda_T+\lambda_D)-(C+\lambda_u+\lambda_d)^2(\lambda_u+\lambda_d) > 0$. For realistic parameters this is trivially satisfied except for very short-term bonds (say a maturity of one month or less). For these bonds, prices are not affected by liquidity shocks unless the shock is large anyway.

paid by both sophisticated and unsophisticated investors decrease in the size of the liquidity shock, and prices facing sophisticated investors decrease more than prices facing unsophisticated investors. The middle graph in Figure 2 illustrates this by showing the difference in midprices between unsophisticated and sophisticated investors. As the previous theorem states this difference increases in the liquidity shock size.

The bottom graph in Figure 2 shows the percentage change in midprice as a function of maturity when 25% of investors experience a liquidity shock. The price impact of a liquidity shock is much higher for a long-maturity bond compared to a short-maturity bond, which illustrates the importance of taking into account maturity when modelling the impact of selling pressure in corporate bonds. The owner of a short-maturity bond with a liquidity need is not willing to sell at a much lower price because the bond will soon mature at par anyway, while the seller of a long-maturity bond accepts a sizeable price discount.

That midprices of sophisticated investors decrease more than midprices of unsophisticated investors is a surprising result. One could have though that a liquidity shock would have a larger impact on prices unsophisticated investors trade at. Indeed, Duffie, Gârleanu, and Pedersen (2007) find that the price impact of a liquidity shock is higher for unsophisticated investors. The key reason for the difference in results is the presence of marketmakers. In their model, there are no marketmakers and investors search for other investors to trade with directly. A liquidity shock decreases the number of buyers, which increases the expected time before a trading partner is found. In my model, sellers and buyers search for a marketmaker to trade with. A liquidity shock does not increase the expected time before a marketmaker is found, but only a fraction of those encounters lead to a trade because there are more sellers than buyers. Thus, in Duffie, Gârleanu, and Pedersen (2007) the price drops because the bargaining position of sellers is worse *and* the expected time for finding a trading partner increases, while my model stresses the bargaining element since the time it takes to find a potential trading partner is the same.

# 3 Estimation Methodology

Corporate bond transactions data only recently became available on a large scale. Since January 2001 FINRA[6] members are required to report their secondary over-the-counter corporate bond transactions through TRACE (Trade Reporting and Compliance Engine). Because of the uncertain benefit to investors of price transparency not all trades reported to TRACE where initially disseminated at the launch of TRACE July 1, 2002. The first research papers using TRACE transactions data focus on the effect of enhanced price transparency and find that dissemination of prices lowered transaction costs for investors (Edwards et al. (2007), Goldstein et al. (2007), and Bessembinder et al. (2006)). The dissemination starts in July 1, 2002 with dissemination of a small subset of trades and from October 1, 2004 all trades are disseminated. Trades must be reported within 15 minutes as of July 1, 2005[7]. TRACE covers all trades in the secondary over-the-counter market for corporate bonds and accounts for more than 99% of the total secondary trading volume in corporate bonds. The only trades not covered by TRACE are trades on NYSE which are mainly small retail trades.

I use a sample of non-callable, non-convertible, straight coupon bullet bonds with maturity less than 30 years from October 1, 2004 to September 30, 2008. For each bond the rating from Moody's, Standard & Poors, and Fitch are downloaded from Bloomberg and an average rating is calculated. The rating from Bloomberg is the rating at the end of the sample period. For a bond to be included in the sample it most have a rating from at least one of the rating agencies. The initial sample of bonds in TRACE is 35,124 and the final sample contains 9,532 bonds. For these bonds I collect the trading history from TRACE covering the period from October 1, 2004 to September 30, 2008 and after filtering out erroneous trades 7,272,454 trades are left. I use the filter proposed by Dick-Nielsen (2009) to filter out error trades.

To estimate the search model outlined in the previous section I need an estimate of roundtrip costs in the dealer market, i.e. the difference between the price at which a dealer sells a bond to a costumer and the price at which a dealer buys a bond from

[6]The Financial Industry Regulatory Authority formerly named National Association of Security Dealers (NASD).

[7]This requirement has gradually been tightened from 1 hour and 15 minutes to 15 minutes. In practice 80% of all transactions are reported within 5 minutes.

a customer. Two main approaches to estimate roundtrip costs exist in the literature. The first is on a given day to average sell prices and subtract average buy prices (Hong and Warga (2000) and Chakravarty and Sarkar (2003)). The second is a regression-based methodology where each transaction price is regressed on a benchmark price and a buy/sell indicator (Bessembinder et al. (2006), Goldstein et al. (2007), and Edwards et al. (2007)). However, both approaches require a buy/sell indicator for each trade, which is not publicly available.

The methodology for estimating roundtrip costs in this paper is based on *unique roundtrip trades* (URT). For a given bond on a given day, if there are exactly 2 or 3 trades for a given volume, I define them to be part of a URT. The intuition is that either 1) a customer sells a bond to a dealer who sells it to another costumer, or 2) a customer sells a bond to a dealer, who sells it to another dealer, who ultimately sells it to a costumer. For a URT the roundtrip cost is defined as the maximal price minus the minimal price. I delete URTs that are zero from the sample. URTs are closely related to Green, Hollifield, and Schürhoff (2007)'s "immediate matches". An "immediate match" is a pair of trades where a buy from a customer is followed by a sale to a customer in the same bond for the same par amount on the same day with no intervening trades in that bond. However, since there is no information about the sides in the transactions in the TRACE database, "immediate trades" cannot be calculated. Also, URTs allow intervening trades, but restrict the number of trades with the same par value to three. In the model in Section 2 dealers do not have an inventory and therefore I restrict the sample to URTs where the trades occur within 15 minutes. Of the 7,272,454 trades in the full sample, 1,593,052 are part of a 15 minutes URT resulting in a 717,826 URTs.

To assess the accuracy of estimated transaction costs when using URTs, Table 1 compares URT estimates of roundtrip costs with those in Edwards et al. (2007) who use a regression-based methodology and have additional buy/sell information that is not publicly available. For this table only the sample period is January 2003-January 2005 in order to match the period in Edwards et al. (2007). Roundtrip costs using unique roundtrip trades executed within 15 minutes are lower than in Edwards et al. (2007). For small trades the costs are about 60% lower while for large trades the costs are around 10% lower. While transaction costs based on 15 minutes URTs likely reflect only search costs, transaction costs for roundtrips for a longer period of time might also reflect inventory costs and/or asymmetric information and thus be higher. Therefore, the table also shows roundtrip costs based on URTs that occur within the same day, and the cost estimates based on these trades are indeed higher and match those in Edwards et al. (2007) fairly well[8]. The roundtrip costs for trade sizes of 100,000 are the same, small trades are underestimated by around 25%, while large trades are overestimated by roughly 10%. The deviations in trading costs can possibly be explained by differences in the sample of bonds. Overall, the numbers in the table suggest that transaction costs based on URTs are reasonable.

One of the predictions of the search model in Section 2 is that investors with the different degrees of sophistication have different roundtrip costs. Thus, it is tempting to find the parameters of the model by fitting actual roundtrip costs to fitted roundtrip costs. However, the model also predicts that midprices of different investor types might vary substantially, so there is important information lost if only roundtrip costs are fitted and instead I fit the model to *demeaned prices*. Any bid or ask prices for a given bond on a given day is demeaned with the average of all bid and ask prices for this bond on this day. That is, if there are $N_{tb}$ URTs on bond $b$ on day $t$, and $A_{tbi}$ is the $i$'th ask price and $B_{tbi}$ the corresponding bid price, the demeaned ask price is defined as $A_{tbi} - \overline{AB}_{tb}$ and demeaned bid price as $B_{tbi} - \overline{AB}_{tb}$ where $\overline{AB}_{tb} = \frac{1}{2N_{tb}} \sum_{i=1}^{N_{tb}} (A_{tbi} + B_{tbmi})$.

For day $t$ and bond $b$ all demeaned bid and ask prices are denoted $P_{tb}^1, P_{tb}^2, ..., P_{tb}^{2N_{tb}-1}$, $P_{tb}^{2N_{tb}}$ (the sorting does not matter). The demeaned fitted prices are denoted $\hat{P}_{tb}^1$, $\hat{P}_{tb}^2, ...$, $\hat{P}_{tb}^{2N_{tb}-1}$, $\hat{P}_{tb}^{2N_{tb}}$ and they are calculated using Theorem 2.1. I assume that each fitting error is normally distributed with zero mean and a standard deviation that depends on the maturity of the bond

$$\begin{aligned} P_{tb}^i - \hat{P}_{tb}^i &\sim N(0, w_{tb}\sigma^2), \\ w_{tb} &= \min(1, T_{tb}^2), \end{aligned}$$

where $T_{tb}$ is the maturity of bond $b$ on day $t$. For a bond with maturity longer than one

[8] Another possible explanation for higher estimated costs for all-day URTs compared to 15 minutes URTs is that all-day URTs are upward biased because prices move during the day. However, Green, Hollifield, and Schürhoff (2007) find that transaction costs for "immediate trades" in the municipal bond market are the same whether or not they control for market movements, so this explanation is unlikely to cause the difference between 15 minutes and all-day URT costs.

year, the standard deviation is increasing in the time-to-maturity. We then have that

$$\epsilon_{tb}^i = \frac{P_{tb}^i - \hat{P}_{tb}^i}{\sqrt{w_{tb}}} \sim N(0, \sigma^2).$$

I define $\Theta$ as a vector with the parameters of the model and assume that errors are independent, such the likelihood function is given as

$$L(\Theta, \sigma|Y) = \prod_{t=1}^{T} \prod_{b=1}^{N_b} \prod_{i=1}^{2N_{tb}} \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-\frac{(\epsilon_{tb}^i)^2}{2\sigma^2})$$

where $N_b$ is the number of bonds in the sample. We therefore have that

$$-2\log L(\Theta, \sigma|Y) = \frac{1}{\sigma^2} \sum_{t=1}^{T} \sum_{b=1}^{N_b} \sum_{i=1}^{2N_{tb}} (\epsilon_{tb}^i)^2 + \sum_{t=1}^{T} \sum_{b=1}^{N_b} \sum_{i=1}^{2N_{tb}} [\log(\sigma^2) + 2\pi] \tag{14}$$

and maximizing the likelihood function therefore amounts to minimizing the sum of squared weighted errors $\sum_{t=1}^{T} \sum_{b=1}^{N_b} \sum_{i=1}^{2N_{tb}} (\epsilon_{tb}^i)^2$. Standard errors are calculated using the outer product of gradients estimator.

The Appendix shows that the sum of squared errors in (14) equals

$$\sum_{t=1}^{T} \sum_{b=1}^{N_b} w_{tb} \Big[ \sum_{i=1}^{N_{tb}} [(A_{tbi} - \overline{A}_{tb}) - (A_{tbi}^M - \overline{A}_{tb}^M)]^2 \tag{15}$$

$$+ \sum_{i=1}^{N_{tb}} [(B_{tbi} - \overline{B}_{tb}) - (B_{tbi}^M - \overline{B}_{tb}^M)]^2 \tag{16}$$

$$+ \frac{N_{tb}}{2} [(\overline{A}_{tb} - \overline{B}_{tb}) - (\overline{A}_{tb}^M - \overline{B}_{tb}^M)]^2 \Big]. \tag{17}$$

where the superscript $M$ refers to fitted prices, $\overline{A}_{tb} = \frac{1}{N_{tb}} \sum_{m=1}^{N_{tb}} A_{tbm}$, and $\overline{B}_{tb} = \frac{1}{N_{tb}} \sum_{m=1}^{N_{tb}} B_{tbm}$. This shows that the fitting procedure of fitting bid and ask prices demeaned by the average midprice can be regarded as fitting ask prices demeaned by average ask price, bid prices demeaned by average bid price, and the average bid-ask spread. In particular, if there is exactly one observation for each bond on each day ($N_{tb} = 1$ for all $t$ and $b$) the expression (15) reduces to $\sum_{t=1}^{T} \sum_{b=1}^{N_b} w_{tb} \frac{1}{2} [(A_{tb} - B_{tb}) - (A_{tb}^M - B_{tb}^M)]$, i.e. fitting bid-ask spreads.

Investors in the search model presented in Section 2 have different speeds with which they meet a market maker. Unsophisticated investors have a low search intensity while sophisticated have a high search intensity. I use trade size as a proxy for investor sophistication. Specifically, there are six investor classes, who differ in their search intensity $\rho$, and they trade in par values of $\$0-5,000$, $\$5,000-20,000$, $\$20,000-100,000$, $\$100,000-500,000$, $\$500,000-1,000,000$, and more than $\$1,000,000$. In addition, I split the bonds up in three rating classes, $AAA-A$, $BBB$, and speculative grade. The bonds differ in their default intensity $\lambda_D$.

There are a number of parameters in the model and many of them are not separately identified. The following parameters are fixed before estimation. The riskless rate is set to $r = 0.05$, which is close to the average 10-year swap rate of 4.94% in the estimation period. The bond has a coupon of 7, and the face value of a bond is set to $F = 100$. The recovery rate on the bond in case of default is set to zero such that $f = 1$ and the bargaining power of the market maker is 0.8. Finally, $\lambda_d = 0.1$ and $\lambda_u = 1$ such that an investor is a "high" type 91% of the time.

# 4 Empirical results

In this section I discuss parameter estimates and the ability of the model to fit actual yield roundtrip costs. Then, I examine time variations in search intensities and shocks to funding liquidity, and finally I assess the impact of search costs on average yields at different maturities.

## 4.1 Parameter estimates and model fit

Table 2 shows the parameter estimates. We see that search intensities increase as investor sophistication increases (proxied by trade size). Roundtrip costs decrease in search intensity according to (13), so increasing search intensities are consistent with results in Chakravarty and Sarkar (2003), Bessembinder, Maxwell, and Venkaraman (2006) Edwards, Harris, and Piwowar (2007), who show that roundtrip costs decrease in trade size. This fact is evident in Panel A of Table 3 where actual and fitted roundtrip costs across trade sizes are shown.

The model slightly underestimates roundtrip costs for the smallest trades and overestimates costs for the largest trades, but overall actual costs matched well. In Panel B we see that roundtrip costs are fitted fairly well across maturity a part from long-maturity bonds where costs are underestimated.

The most unsophisticated investors (trading in sizes between 0 and $\$5,000$) have a search intensity of 31. This implies that they need eight business days on average before they find a market maker with whom to trade with. This can be viewed as the time it takes a non-professional to learn how to trade in the corporate bond market and keep up-to-date about information relevant for trading. The most sophisticated investors (trading sizes of more than $\$1,000,000$) have a search intensity of 171 implying that it takes 1-2 days to complete trades of large size.

The parameter $\tilde{q}$ is estimated to be 0.68, which implies that in a trade intermediated by a market maker the gains of trade is twice as large for the seller compared to the buyer. Note that $\tilde{q}$ does not appear in the determination of bid-ask spreads in equation (13). Rather, the identification of $\tilde{q}$ is through the relation between bid-ask spreads of different investors. In order to understand the effect of $\tilde{q}$ consider $\tilde{q} = 1$. In this case, both retail and institutional buyers are almost indifferent between trading or not because the interdealer price is high, and they trade at similar prices. In contrast, the gains of trade are large for sellers. The "threat" of buyers to cut off negotiations and let a seller wait until he meets another counterparty, is particularly strong for retail investors and they accept a lower price compared to the institutional investor. Overall, this means that mid-prices of retail investors are lower than mid-prices of institutional investors. If $\tilde{q} = 0$ the reverse is the case and midprices of retail investors are higher than those of institutional investors. Since $\tilde{q} = 0.68$ institutional midprices are higher than retail midprices on average.

The liquid market for credit default swaps might be a reason for a $\tilde{q}$ above 0.5. There is an approximate arbitrage relation between corporate bonds and credit default swaps: one can approximately create a bond with no default risk by buying a corporate bond and insuring against default by buying protection through a credit default swap (Duffie (1999), Longstaff, Mithal, and Neis (2005), and Blanco, Brennan, and Marsh (2005)). If corporate bond prices are low relative to credit default swaps, one can buy the bond and buy protection through a credit default swap contract and earn an abnormal profit. If prices are high relative to credit default swaps, one needs to short the bond and sell protection through a credit default swap. It can be difficult and expensive to short the bond as shown in Nashikkar and Pedersen (2007), so the arbitrage is easer to carry out when corporate bond prices are low compared to when they are high.

The following calculations provide an estimate of the additional cost due to search that investors in the corporate bond market incur compared to that of the Treasury market. The average maturity in the data sample is 5.5 years, so a 5-year bond is the most representative bond for the corporate market. An estimate of the average bid-ask spread as a percentage of par value of a 5-year bond in the Treasury market is 0.000122 according to Fleming (2003). For an average investor, i.e. an investor with an average search intensity, the corresponding estimate for a 5-year bond in the corporate bond market is 0.00289 according to the parameter estimates and equation (13). Thus, an estimate of the cost of search on a roundtrip in the corporate bond market relative to the Treasury market is 0.00277. The yearly trading volume in the corporate bond market was $6196.5 billion in 2007, so an estimate of the additional yearly costs investors bear in the corporate bond market compared to the Treasury market is $6196.5*0.00277 =$14.11 billion[9].

## 4.2 Time variation in trading costs

The parameter estimates of the calibration reflect average market conditions over the estimation period. However, the period has seen several significant events in the corporate bond market, evidenced for example by the fact that the 10-year spread between highly rated AAA corporate bonds and Treasury bonds has ranged from a low of 64 basis points to a high of 200 basis[10]. To examine the importance of shifting conditions in the market, I estimate the time-variation in search intensities as follows. For month $m$, I multiply all search intensities by $c_m$. Based on all the observations in that month, I find the maximum-likelihood estimate (according to the likelihood procedure explained earlier) of $c_m$ holding all other parameters fixed. I do this for every month in the sample.

Search intensities are determined primarily by the size of roundtrip costs, so variations over time in search intensities measure the cost of buying and selling bonds. Since conditions in the corporate bond market might affect retail and institutional investors differently, I estimate time variation in search intensities separately for each investor type. That is, I split the data sample up in trades of less than $\$100,000$ (retail) and $\$100,000$ or more (institutional) and carry out the aforementioned time series estimation on each data set.

[9]Average daily volume was $24.3 billion according to footnote 1, so yearly was 255*$24.3 billion.
[10]The yield spread is basis on monthly data from the Federal Reserve's H.15 data. The AAA yield is "Moody's seasoned Aaa" while Treasury yield is the 10-year constant maturity rate.

Time variations in intensities are shown in Figure 3. We see that the scaling factor for search intensities of retail investors is increasing steadily during the estimation period from 0.75 in October 2005 to 1.05 in October 2008. The increase in search intensities implies a decrease in roundtrip costs. For example, an investor trading a 10-year bond in small lots of $5,000 or less has seen a decrease in roundtrip costs from $0.86 to $0.72 per $100 face value. Just as retail investors, institutional investors see their trading costs go down from October 2005 to March 2007, but then see a drastic increase from March 2007 to the end of the sample in October 2008. The search intensity scaling factor goes from 1.06 in October 2005 to 1.44 in March 2007 and ends in 0.69 in October 2008. In dollar terms, an institutional investor trading a notional of $1,000,000 in a 10-year bond had roundtrip costs of $14,762 in October 2005, $10,962 in March 2007, and $22,281 in October 2008. Overall, the evidence in Figure 3 suggests that after the onset of the subprime crisis, small trades can be carried out at normal roundtrip costs, while larger trades are harder to execute and therefore reflect larger roundtrip costs.

The findings are in line with concerns being voiced that it has become more difficult to execute large trades. National Association Of Securities Dealers write that "[m]ore recently, there have been reports that dealers may be abandoning the corporate bond market as a result of TRACE and subsequent reduced profitability" and "concerns have also been raised that transparency has made it difficult for large trades to get done and that liquidity in the largest trade sizes has declined" (NASD (2006)). Also, "it may have previously been possible to complete a sizeable bond purchase with a single phone call to dealer...the post-TRACE environment may involve communications with multiple dealers and delays as the dealers search for counterparties." (Bessembinder and Maxwell (2008)).

## 4.3 Liquidity shocks

The results in Theorem 2.3 show that a liquidity shock to sellers does not affect bid-ask spreads. The time variation in search costs estimated in the previous section therefore does not reveal whether there has been shocks to sellers or not. Instead, shocks to funding liquidity is detected through a higher than usual difference between retail mid-prices and institutional mid-prices. To estimate possible shocks to the number of sellers, I do the following. For month $m$, I assume a liquidity shock of size $s_m$ occurs as defined in Definition 2.1. Holding all other parameters fixed including the value of $c_m$ estimated in the previous section, I estimate $s_m$ by maximum likelihood (prices are calculated according to Theorem 2.2). I do this for every month in the sample. Since there is a sharp distinction between investing in investment grade securities and speculative grade securities, I divide the sample into those two groups.

Figure 4 graphs the estimated liquidity shocks for investment grade and speculative grade investors. In the first part of the sample period, there is one shock occuring. The shock occurs only in the speculative grade segment and essentially starts in March 2005, peaks in May 2005, and thereafter disappears quickly. The shock can be contributed to events in the automobile companies GM and Ford. The rating of bonds is registered at the end of the sample, and GM and Ford bonds have a speculative grade rating, which explains why the shock happens in the speculative grade segment. Since GM issued a steep profit warning in March 16, 2005, GM and Ford had been trading at or near junk levels. Downgrade was imminent and many insurance companies, pension funds, and other investment funds were restricted from investing in junk bonds and were forced to liquidate GM/Ford bonds. Acharya, Schaefer, and Zhang (2008) find an increasing imbalance between bid and offer quotes on GM and Ford bonds after the profit warning in March lasting until a month after the downgrade to junk bond status in May 5. Whereas the largest shocks appear around the downgrade, there are smaller shocks appearing already in the beginning of the sample period in October 2004 indicating moderate selling pressure for an extended period preceding the downgrade. As BIS (2005) write "the downgrade had long been anticipated and so asset managers had ample opportunity to adjust their portfolios. Since mid-2003, the auto makers' spreads had been trading closer to speculative grade issuers than those on other BBB-rated issuers."

The second period with a large number of forced sellers according to the estimation results coincides with the subprime crisis. Interestingly, the first sign of a liquidity shock appears already in April 2007 in investment grade bonds while speculative grade bonds see their first shock as late as in November 2007. The largest shock for both investment grade and speculative grade bonds happens in March 2008. BIS (2008a) writes that "(t)urmoil in credit markets deepened in early March...tightening repo haircuts caused a number of hedge funds and other leveraged investors to unwind existing positions. As a result, concerns about a cascade of margin calls and forced asset sales accelerated the ongoing investor withdrawal from various financial markets. In the process, spreads on even the most highly rated assets reached unusually wide levels, with market liquidity disappearing across most fixed income markets." A liquidity squeeze on Bear Sterns caused a take-over by JPMorgan on March 17. The Federal Reserve cut the policy rate by 75 basis points, and "(t)hese developments appeared to herald a turning point in the market...with investors increasingly adopting the view that various central bank initiatives aimed at reliquifying previously dysfunctional markets were gradually gaining traction" (BIS (2008a)). According to the model, any sign of liquidity shocks almost disappeared in May 2008 with the percentage of bond owners with a sudden need to sell down to around 1% for both investment grade and speculative grade investors. According to BIS (2008a), "(b)y the end of the period in late May, the process of disorderly deleveraging had come to a halt, giving way to more orderly credit market conditions. Market liquidity had improved and risk appetite increased, luring investors back into the market". However, this rebound of the corporate bond market was short-lived and the model-implied liquidity shocks peak again in the two final months in the sample period, August and September 2008. As BIS (2008b) writes "in late August, credit spreads had drifted upwards once again...Despite an aggregate $503 billion of assets written down by banks and brokerages since the start of the credit crisis in 2007, further writedowns and outright asset disposals were thus seen as continuing over the coming months, adding to existing capital constraints and related funding needs." On September 15 Lehman Brothers filed for bankruptcy, one of the biggest credit events in history, and a trigger for a new and intensified stage of the credit crisis.

Overall, this section shows that model-implied liquidity shocks coincide with main events in the corporate bond market during the sample period.

## 4.4 Yield premia in the OTC corporate bond market

One of the most widely employed frameworks of credit risk, structural models, was developed in the seminal work of Merton (1974). Structural models take as given the dynamics of the value of a firm and value corporate bonds as contingent claims on the firm value. In structural models the spread between the yield on a corporate bond and the riskless rate goes to zero as maturities shortens. However, yield spreads are typically positive, also at very short maturities, and this has given rise to the "credit spread puzzle", namely that corporate yield spreads, particularly at very short maturities, are too high to be explained by the corporate bond issuer's default risk (see for example Huang and Huang (2003) and Chen, Collin-Dufresne, and Goldstein (2008))[11]. This paper offers an explanation for the credit risk puzzle, namely costs due to search frictions and occasional selling pressure[12].

I define the search premium for an investor as the midyield paid by this investor minus the yield of an investor who can instantly find a trading partner ($\rho = \infty$) in which case the bid-ask spread is zero. This mimics a trade in the corporate bond market versus a trade in the ultra-liquid Treasury market. I do this for an "average" corporate bond investor, where the search intensity is the average of all the estimated search intensities in Table 2. For the same "average" investor I define the selling pressure premium as the midyield on trades occuring under a liquidity shock of 7.5% (largely matching the average liquidity shock for both investment grade and speculative grade investors during the sample) minus the midyield on trades occuring in the absence of a liquidity shock.

Figure 5 graphs the term structure of search premia and selling pressure premia. The figure shows that search costs affect primarily the short end of the yield curve with a premium of almost 100 basis points for bonds with very short maturities (less than say two weeks). For bonds with longer maturities (a year or more) the effect of search costs is very modest and in the single-digit range. Thus, although search-induced bid-ask spreads decrease as maturity decrease, the effect of search costs *on yields* increases as maturity decreases. Covitz and Downing (2007) examine very short-term spreads in the commercial paper market for the roles of credit risk and liquidity. The average trade size in their data set is USD15 million, so the trades are a magnitude larger than those examined in this paper. They find that trade size plays a role in the determination of commercial paper spreads and that the effect of trade size decreases with maturity, consistent with the results here. Figure 5 also shows that the average effect of occasional selling pressure increases moderately as maturities lengthens. For a ten-year bond the

[11]Extensions to structural models that can explain the short-maturity credit spread puzzle include jumps in firm value (Zhou (2001)) and incomplete accounting information (Duffie and Lando (2001)).

[12]The model in this paper is related to reduced-form models of credit risk, where there is an intensity process governing the risk of default. Thus, it does nor predict a near zero contribution of default risk to spreads at very short maturities as structural models, but nevertheless the implications of search costs can be examined in the model.

effect is around 40 basis points. In contrast, there is no effect of selling pressure at very short maturities.

Search costs and selling pressure provide a rich explanation for the credit spread puzzle. Search costs dominate short-term premia while selling pressure dominates long-term premia, and the term structure of total premia can obtain a variety shapes depending on conditions in the corporate bond market - such as downward-sloping, upward-sloping, and hump-shaped - and long-term premia and short-term premia can move independently of each other. The magnitudes of premia are comparable to the non-default component of corporate bond spreads found in other papers. For example, both Huang and Huang (2003) and Longstaff, Mithal, and Neis (2005) find the average non-default component of the 5-year AAA-Treasury spread to be 50-55 basis points. I find it to be around 40 basis points. Although such comparisons should be interpreted with care due to differences in sample periods and estimation methodologies, it does show that the estimated premia in this paper is of a size that explains the credit spread puzzle.

# 5 Conclusion

The corporate bond market is an over-the-counter market where investors search for counterparties and once there is a match, prices are determined through bargaining. I set up a model that captures the search-and-bargaining features, and a key result in the model is that in a market with many sellers, midprices of institutional investors are lower than those of retail investors. I estimate model parameters using transaction data from October 2004 to September 2008 and find that model-implied periods with a large number of sellers coincide with important events in the corporate bond market. I also find that trading costs have increased strongly for institutional investors during the subprime crisis, a period that according to the model has seen a large number of forced sellers.

Since midprices are higher for small trades compared to large trades, it might be optimal for investors to trade in smaller trade sizes after the onset of the crisis, and Dick-Nielsen, Feldhütter, and Lando (2008) find support for this. Taking into account liquidity shocks and investors' response to shocks in risk management might lead to tighter risk management, which in turn can lead to a more illiquid market as in Gârleanu and Pedersen (2007). Therefore, modelling liquidity shocks and the optimal selling behavior in response to liquidity shocks is an interesting subject for future research with important implications for risk management and the severity of liquidity shocks.

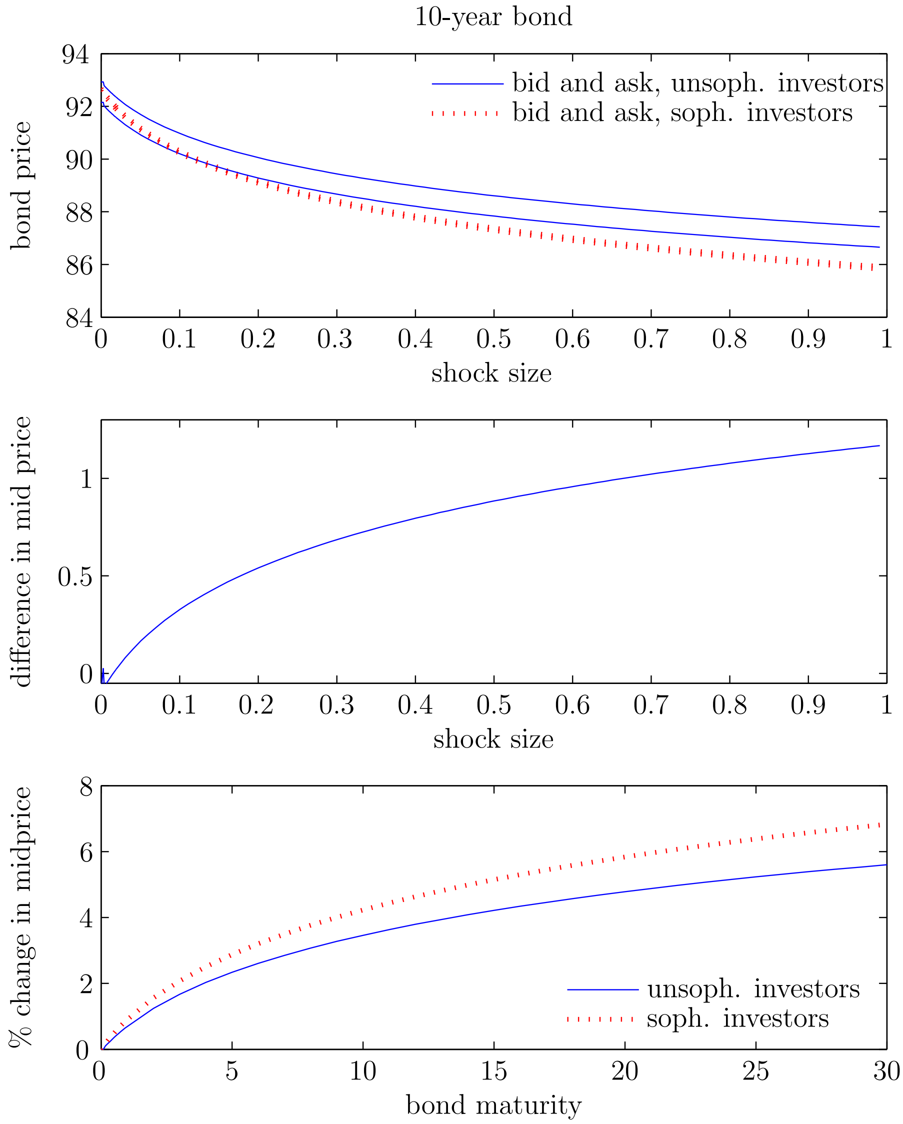


**Figure 2:** *Prices and liquidity shocks.* The top graph shows bid and ask prices for a 10-year bond for an unsophisticated and sophisticated investor as a function of size of liquidity shock. The middle graph shows the difference in midprice for an unsophisticated investor and midprice for a sophisticated investor for the same 30-year bond. The bottom graph shows the percentage change in midprice for an unsophisticated and sophisticated investor as a function of bond maturity when 25% of investors experience a liquidity shock. The parameters are $\rho_{unsoph} = 50, \rho_{soph} = 300, \lambda_u = 1, \lambda_d = 0.1, \delta = 7, C = 7, \lambda_D = 0.01, r = 0.05, F = 100, f = 1, z = 0.8, \tilde{q} = 0.65$.

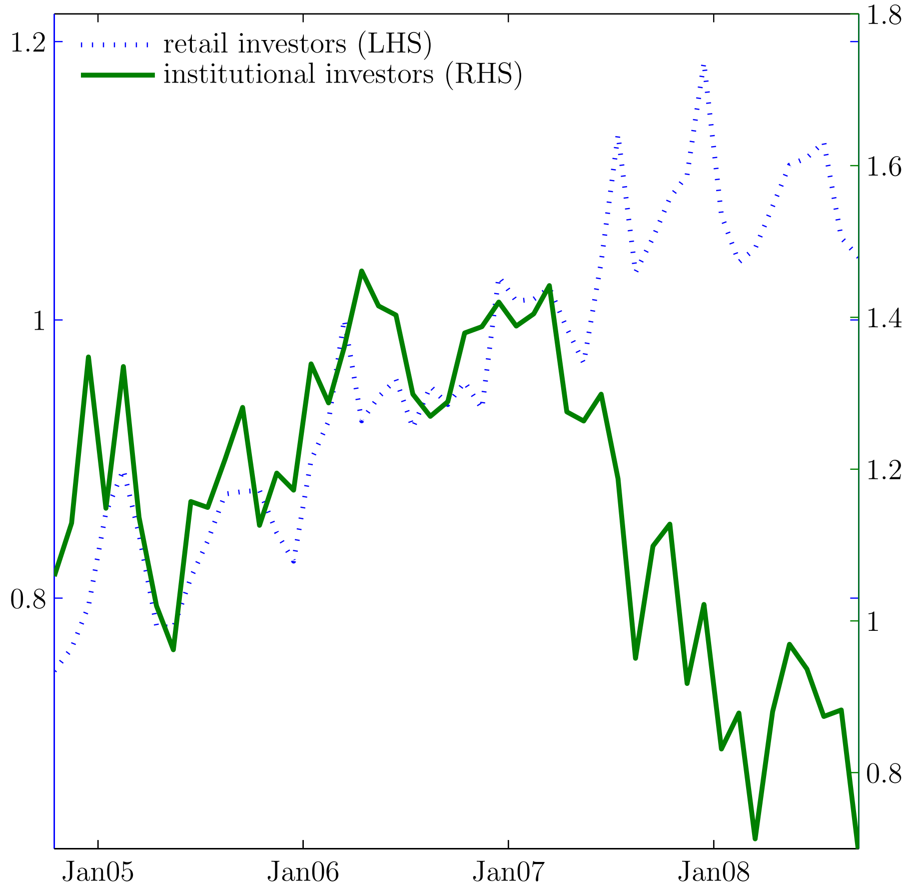


**Figure 3:** *Time-variation in ease of finding a counterparty.* This graph shows the monthly time-variation in search intensity $\rho$ for institutional and retail investors. A higher search intensity implies lower bid-ask spreads. Retail investors are assumed to trade par amounts less than \$100,000 while institutional investors are assumed to trade \$100,000 or more.

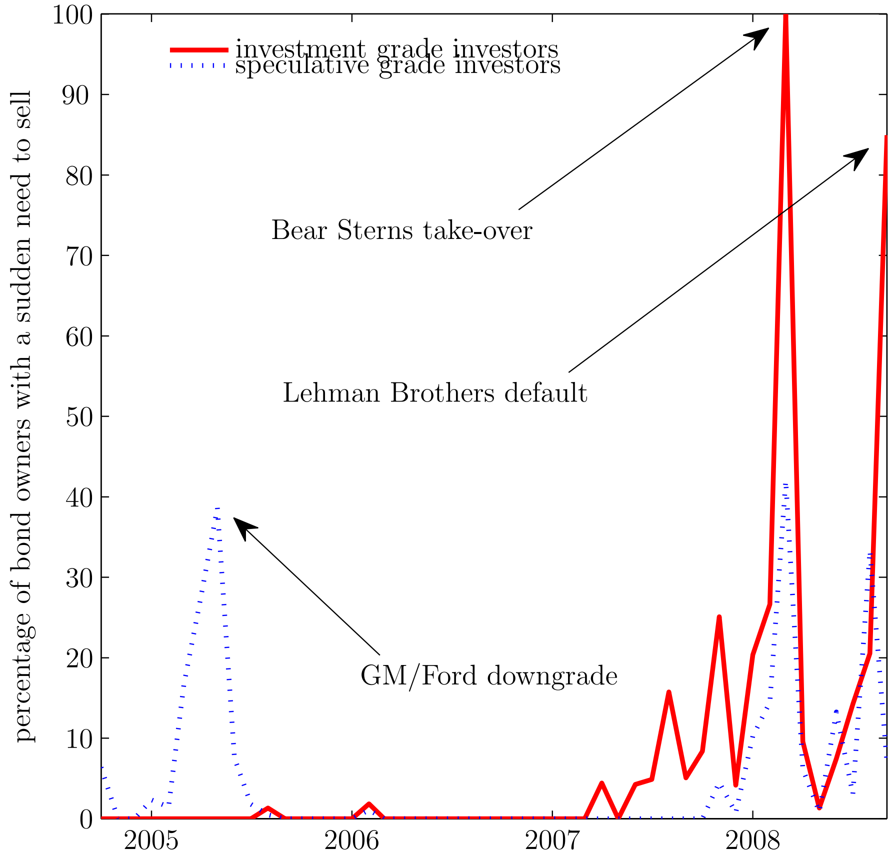


**Figure 4:** *Fraction of bond owners experiencing a negative shock in their funding liquidity.* The number of investors with high liquidity in steady state are shocked, bid and ask yields immediately after the shock are derived, and the most likely shock is found as explained in the text. This is done for each month in the sample. The sample is split up into investment grade bonds and speculative grade bonds.

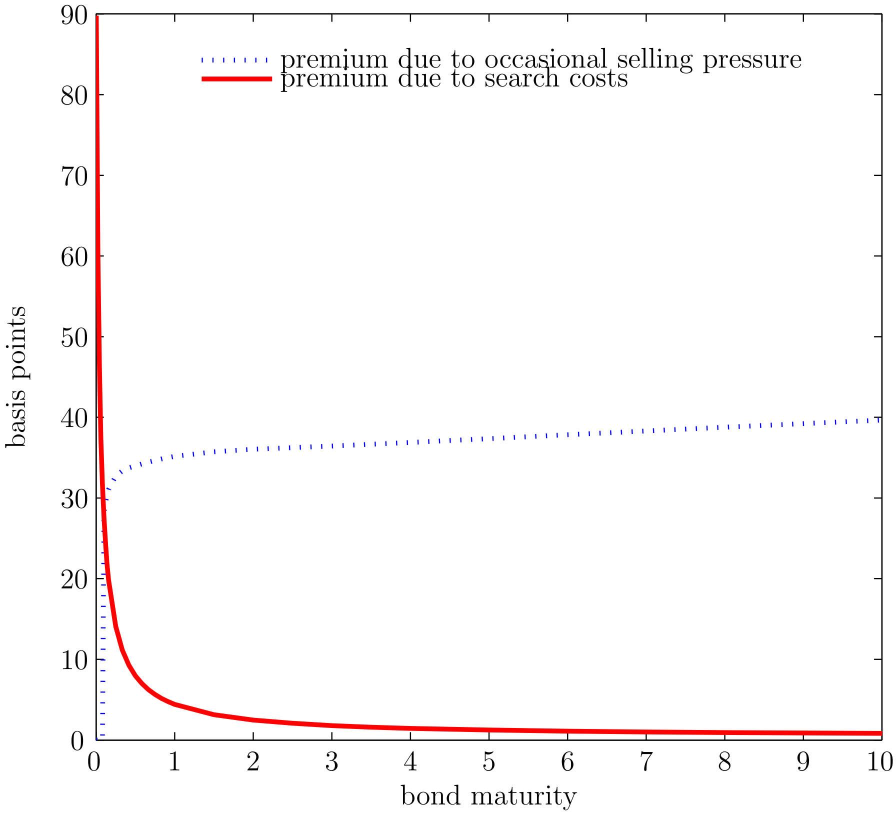


**Figure 5:** *Premium in yields due to search costs and occasional selling pressures.* This graph shows the premium in yields across bond maturity due to search costs and occasional selling pressures. The search premium for an investor for a given maturity is defined as the average yield of a buy and sell transaction for this investor minus the average yield of a buy and sell transaction for an investor that can instantly find a trading partner. In the latter case, the buy and sell yields are identical. The premium due to selling pressure for an investor for a given maturity is defined as the average yield of a buy and sell transaction for this investor right after a shock occurs minus the average yield of a buy and sell transaction in the absence of a shock. Investors are assumed to have a search intensity of $\rho = 135$ and the number of investors being hit by a liquidity shock is 7.5%.

| Trade size | $5K$ | 10K | 20K | 50K | 100K | 200K | 500K |
|---|---|---|---|---|---|---|---|
| Edwards, Harris, and Piwowar (2007) | 1.50 | 1.42 | 1.24 | 0.92 | 0.68 | 0.48 | 0.28 |
| | | | | | | | |
| Unique Roundtrip Trades (15mins) | 0.91 | 0.82 | 0.78 | 0.61 | 0.56 | 0.44 | 0.25 |
| Unique Roundtrip Trades (15mins), observations | 8,385 | 14,819 | 9,943 | 10,114 | 6,566 | 2,189 | 2,907 |
| | | | | | | | |
| Unique Roundtrip Trades (all day) | 1.14 | 1.10 | 1.04 | 0.79 | 0.68 | 0.53 | 0.31 |
| Unique Roundtrip Trades (all day), observations | 19,017 | 32,298 | 20,470 | 23,454 | 15,678 | 4,829 | 7,147 |

**Table 1:** *Roundtrip costs.* This table compares roundtrip costs using this paper's *unique roundtrip trade* measure with those in Edwards, Harris, and Piwowar (2007)'s Table IV. A trade is defined as a unique roundtrip if for a given bond on a given date 2 or 3 trades with a given trade size occurs. The roundtrip cost is the largest minus the smallest trade price. The table shows the roundtrip costs for both unique roundtrips that occur during a trading day and those that occur within 15 minutes. The URT roundtrip estimates in this table is based on the sample period January 2003-January 2005 which matches that in Edwards, Harris, and Piwowar (2007).

| $\tilde{q}$ | $\rho_1$ | $\rho_2$ | $\rho_3$ | $\rho_4$ | $\rho_5$ | $\rho_6$ |
|---|---|---|---|---|---|---|
| 0.678 | 31 | 46 | 55 | 92 | 144 | 173 |
| (0.016) | (0.3) | (0.4) | (0.5) | (2.4) | (14.9) | (9.9) |

**Table 2:** *Parameter estimates for a search model for the corporate bond market.* This table shows the estimated parameters of the search model presented in Section 2. The model parameters are estimated by maximum likelihood and standard errors are calculated using the outer product of gradients estimator. Corporate bond data used in estimation are actual transactions from TRACE for the period October 1, 2004 to September 30, 2008.

Panel A: Trade size

| | 0-5K | 6-10K | 11-20K | 21-50K | 51-100K | 101-200K | 201-500K | 501-1000K | >1000K |
|---|---|---|---|---|---|---|---|---|---|
| fitted (in bps) | 70.8 | 51.8 | 51.9 | 44.2 | 44.2 | 27.9 | 27.9 | 16.7 | 15.5 |
| | (0.0) | (0.0) | (0.0) | (0.0) | (0.0) | (0.0) | (0.0) | (0.0) | (0.0) |
| actual (in bps) | 80.1 | 65.2 | 65.5 | 59.5 | 51.5 | 38.9 | 26.5 | 17.2 | 10.4 |
| | (0.3) | (0.2) | (0.2) | (0.1) | (0.2) | (0.3) | (0.2) | (0.2) | (0.1) |

Panel B: Maturity

| | 0-2m | 2m-4m | 4-6m | 6m-1y | 1-3y | 3-5y | 5-30y |
|---|---|---|---|---|---|---|---|
| fitted (in bps) | 25.4 | 35.8 | 40.3 | 43.7 | 47.0 | 46.9 | 47.5 |
| | (0.1) | (0.1) | (0.1) | (0.0) | (0.0) | (0.0) | (0.0) |
| actual (in bps) | 22.3 | 24.3 | 27.2 | 31.8 | 44.4 | 54.8 | 77.6 |
| | (0.5) | (0.4) | (0.3) | (0.2) | (0.1) | (0.1) | (0.2) |

**Table 3:** *Estimated round-trip costs.* This table reports the fitted roundtrip costs in basis points of par value for the search model presented in Section 2. Parameters in the model are those in Table 2. Below the fitted roundtrip costs are actual roundtrip costs. Corporate bond data used in estimation as well as in calculation of actual roundtrip costs are transactions from TRACE for the period October 1, 2004 to September 30, 2008.

# A Equilibrium Allocations

The rate of change of mass of owners is

$$\dot{\mu}_{lo}(t) = -\rho\mu_m(t) \quad -(\lambda_T+\lambda_D)\mu_{lo}(t) - \lambda_u\mu_{lo}(t) + \lambda_d\mu_{ho}(t) \tag{18}$$

$$\dot{\mu}_{ho}(t) = \rho\mu_{hn}(t) \quad -(\lambda_T+\lambda_D)\mu_{ho}(t) + \lambda_u\mu_{lo}(t) - \lambda_d\mu_{ho}(t) \tag{19}$$

$$\dot{\mu}_{hn}(t) = -\rho\mu_{hn}(t) \quad +(\lambda_T+\lambda_D)\mu_{ho}(t) + \lambda_u\mu_{ln}(t) - \lambda_d\mu_{hn}(t) \tag{20}$$

$$\dot{\mu}_{ln}(t) = \rho\mu_m(t) \quad +(\lambda_T+\lambda_D)\mu_{lo}(t) - \lambda_u\mu_{ln}(t) + \lambda_d\mu_{hn}(t) \tag{21}$$

where $\mu_m(t) = \min\{\mu_{hn}(t), \mu_{lo}(t)\}$.

If $\mu_m = \mu_{hn}$ in steady state then the sum of (18) and (19) yields $(\lambda_T+\lambda_D)(\mu_{lo}+\mu_{ho}) = 0$ which cannot be the case, so $\mu_m = \mu_{lo}$ in steady state. Inserting $\mu_{hn} = 1-(\mu_{ho}+\mu_{ln}+\mu_{lo})$ into (21) and letting $\dot{\mu}_{ln}(t) = 0$ yields

$$-\lambda_d = (\rho+\lambda_T+\lambda_D-\lambda_d)\mu_{lo} - (\lambda_u+\lambda_d)\mu_{ln} - \lambda_d\mu_{ho}$$

so in steady state we have

$$\begin{pmatrix} \mu_{lo} \\ \mu_{ho} \\ \mu_{hn} \\ \mu_{ln} \end{pmatrix} = A^{-1} \begin{pmatrix} 0 \\ 0 \\ 0 \\ -\lambda_d \end{pmatrix}$$

where

$$A = \begin{pmatrix} -(\rho+\lambda_T+\lambda_D+\lambda_u) & \lambda_d & 0 & 0 \\ \lambda_u & -(\lambda_T+\lambda_D+\lambda_d) & \rho & 0 \\ 0 & \lambda_T+\lambda_D & -(\lambda_d+\rho) & \lambda_u \\ \rho+\lambda_T+\lambda_D-\lambda_d & -\lambda_d & 0 & -(\lambda_u+\lambda_d) \end{pmatrix}.$$

# B Equilibrium prices

This Appendix derives the value functions stated in the text and the prices stated in Theorem 2.1.

From (5) and (6) we have that lifetime utility is $W(t) + V_{\sigma(t)}$ where

$$V_{\sigma(t)}(t) = \sup_{\theta} E_t\Big[\int_t^{\infty} e^{-r(t-s)}\theta_s(C - \delta 1_{\{\sigma^\theta(s)=lo\}})ds - e^{-r(s-t)}\hat{P}_s d\theta_s\Big].$$

In order to calculate $V_\sigma$ and the bid/ask prices, we consider a particular agent and a particular time $t$. Let $\tau_l$ be the next stopping time at which the agent's type changes, $\tau_m$ the next time a market maker is met, (in case of an owner) $\tau_T$ the time at which the bond matures and $\tau_D$ the time at which the bond defaults. Furthermore, let $\overline{\tau} = min\{\tau_l, \tau_m\}$, $\tilde{\tau} = min\{\tau_l, \tau_T, \tau_D\}$, and $\tau = min\{\tau_l, \tau_m, \tau_T, \tau_D\}$. Then

$$\begin{aligned}
V_{ln}(t) &= E_t[e^{-r(\tau_l - t)}V_{hn}(\tau_l)] \\
V_{hn}(t) &= E_t[e^{-r(\tau_l - t)}V_{ln}(\tau_l)1_{\{\tau_l = \overline{\tau}\}} + e^{-r(\tau_m - \tau)}(V_{ho}(\tau_m) - A_{\tau_m})1_{\{\tau_m = \overline{\tau}\}}] \\
V_{lo}(t) &= E_t[\int_t^\tau e^{-r(u-t)}(C - \delta)du + e^{-r(\tau_l - t)}V_{ho}(\tau_l)1_{\{\tau_l = \tau\}} \\
&\quad + e^{-r(\tau_m - t)}(V_{ln}(\tau_m) + B_{\tau_m})1_{\{\tau_m = \tau\}} \\
&\quad + e^{-r(\tau_T - t)}(V_{ln}(\tau_T) + F)1_{\{\tau_T = \tau\}}] \\
&\quad + e^{-r(\tau_D - t)}(V_{ln}(\tau_D) + (1-f)F)1_{\{\tau_D = \tau\}}] \\
V_{ho}(t) &= E_t[\int_t^{\tilde{\tau}} Ce^{-r(u-t)}du + e^{-r(\tau_l - t)}V_{lo}(\tau_l)1_{\{\tau_l = \tilde{\tau}\}} \\
&\quad + e^{-r(\tau_T - t)}(V_{hn}(\tau_T) + F)1_{\{\tau_T = \tilde{\tau}\}}] \\
&\quad + e^{-r(\tau_D - t)}(V_{hn}(\tau_D) + (1-f)F)1_{\{\tau_D = \tilde{\tau}\}}]
\end{aligned}$$

Suppressing the dependence on time, the value functions satisfy the (HJB) equations

$$\dot{V}_{ln} = rV_{ln} - \lambda_u(V_{hn} - V_{ln}) \tag{22}$$

$$\dot{V}_{hn} = rV_{hn} - \lambda_d(V_{ln} - V_{hn}) - \rho(V_{ho} - V_{hn} - A) \tag{23}$$

$$\begin{aligned}
\dot{V}_{lo} &= rV_{lo} - \lambda_u(V_{ho} - V_{lo}) - \rho(B + V_{ln} - V_{lo}) - \lambda_T(F + V_{ln} - V_{lo}) \\
&\quad - \lambda_D((1-f)F + V_{ln} - V_{lo}) - (C - \delta)
\end{aligned} \tag{24}$$

$$\begin{aligned}
\dot{V}_{ho} &= rV_{ho} - \lambda_d(V_{lo} - V_{ho}) - \lambda_T(F + V_{hn} - V_{ho}) \\
&\quad - \lambda_D((1-f)F + V_{hn} - V_{ho}) - C
\end{aligned} \tag{25}$$

To see this, we explicitly derive equation (25) and note that the derivation of (22)-(24)

is very similar. We have that

$$
\begin{aligned}
E_t[\int_t^{\tilde{\tau}} e^{-r(u-t)}du] &= \int_t^{\infty}\int_0^{\tilde{\tau}-t} e^{-ru}dud\tilde{\tau} \\
&= \int_t^{\infty}\frac{1}{r}(1-e^{-r(\tilde{\tau}-t)})d\tilde{\tau} \\
&= \int_0^{\infty}\frac{1}{r}(1-e^{-r\tilde{\tau}})d\tilde{\tau} \\
&= \int_0^{\infty}\frac{1}{r}(1-e^{-rx})(\lambda_d+\lambda_T+\lambda_D)e^{-x(\lambda_d+\lambda_T+\lambda_D)}dx \\
&= \frac{\lambda_d+\lambda_T+\lambda_D}{r}\Big[\frac{-1}{\lambda_d+\lambda_T+\lambda_D}e^{-x(\lambda_d+\lambda_T+\lambda_D)} \\
&\quad +\frac{1}{r+\lambda_d+\lambda_T+\lambda_D}e^{-x(r+\lambda_d+\lambda_T+\lambda_D)}\Big]_0^{\infty} \\
&= \frac{\lambda_d+\lambda_T+\lambda_D}{r}(\frac{1}{\lambda_d+\lambda_T+\lambda_D}-\frac{1}{r+\lambda_d+\lambda_T+\lambda_D}) \\
&= \frac{1}{r+\lambda_d+\lambda_T+\lambda_D}
\end{aligned}
$$

and for $\tau = min\{\tau_1, \tau_2\}$ that

$$
\begin{aligned}
E_t[e^{-r(\tau_1-t)}1_{\{\tau_1=\tau\}}V(\tau_1)] &= \int_t^{\infty}\int_t^{\infty} e^{-r(x-t)}V(x)\lambda_1 e^{-\lambda_1(x-t)}\lambda_2 e^{-\lambda_2(y-t)}1_{\{x<y\}}dxdy \\
&= \int_t^{\infty}\int_x^{\infty} e^{-r(x-t)}V(x)\lambda_1 e^{-\lambda_1(x-t)}\lambda_2 e^{-\lambda_2(y-t)}dydx \\
&= \int_t^{\infty} e^{-r(x-t)}V(x)\lambda_1 e^{-\lambda_1(x-t)}[-e^{-\lambda_2(y-t)}]_x^{\infty}dx \\
&= \int_t^{\infty} e^{-r(x-t)}V(x)\lambda_1 e^{-\lambda_1(x-t)}e^{-\lambda_2(x-t)}dx \\
&= \int_t^{\infty} \lambda_1 e^{-(r+\lambda_1+\lambda_2)(x-t)}V(x)dx
\end{aligned}
$$

Such that[13]

$$
\begin{aligned}
\dot{V}_{ho} &= \frac{\partial}{\partial t}E_t[\int_t^{\tilde{\tau}} Ce^{-r(u-t)}du + e^{-r(\tau_l-t)}V_{lo}1_{\{\tau_l=\tau\}} \\
&\quad + e^{-r(\tau_T-t)}(V_{hn}+F)1_{\{\tau_T=\tau\}} + e^{-r(\tau_D-t)}(V_{hn}+(1-f)F)1_{\{\tau_D=\tau\}}] \\
&= \frac{\partial}{\partial t}E_t[e^{-r(\tau_l-t)}V_{lo}1_{\{\tau_l=\tau\}} + e^{-r(\tau_T-t)}(V_{hn}+F)1_{\{\tau_T=\tau\}} \\
&\quad + e^{-r(\tau_D-t)}(V_{hn}+(1-f)F)1_{\{\tau_D=\tau\}}] \\
&= \int_t^\infty \lambda_d(r+\lambda_d+\lambda_T+\lambda_D)e^{-(r+\lambda_d+\lambda_T+\lambda_D)(x-t)}V_{lo}dx - \lambda_d V_{lo} \\
&\quad + \int_t^\infty \lambda_T(r+\lambda_d+\lambda_T+\lambda_D)e^{-(r+\lambda_d+\lambda_T+\lambda_D)(x-t)}(V_{hn}+F)dx - \lambda_T(V_{hn}+F) \\
&\quad + \int_t^\infty \lambda_D(r+\lambda_d+\lambda_T+\lambda_D)e^{-(r+\lambda_d+\lambda_T+\lambda_D)(x-t)}(V_{hn}+(1-f)F)dx \\
&\quad - \lambda_D(V_{hn}+(1-f)F) \\
&= (r+\lambda_d+\lambda_T+\lambda_D)E_t[e^{-r(\tau_l-t)}V_{lo}1_{\{\tau_l=\tilde{\tau}\}}] - \lambda_d V_{lo} \\
&\quad + (r+\lambda_d+\lambda_T+\lambda_D)E_t[e^{-r(\tau_T-t)}(V_{hn}+F)1_{\{\tau_T=\tilde{\tau}\}}] - \lambda_T(V_{hn}+F) \\
&\quad + (r+\lambda_d+\lambda_T+\lambda_D)E_t[e^{-r(\tau_D-t)}(V_{hn}+(1-f)F)1_{\{\tau_D=\tilde{\tau}\}}] \\
&\quad - \lambda_D(V_{hn}+(1-f)F) \\
&= (r+\lambda_d+\lambda_T+\lambda_D)[V_{ho} - \frac{C}{r+\lambda_d+\lambda_T+\lambda_D}] - \lambda_d V_{lo} - \lambda_T(V_{hn}+F) \\
&\quad + \lambda_D(V_{hn}+(1-f)F) \\
&= (r+\lambda_d+\lambda_T+\lambda_D)V_{ho} - \lambda_d V_{lo} - \lambda_T(V_{hn}+F) - \lambda_D(V_{hn}+(1-f)F) - C.
\end{aligned}
$$

In steady state $\dot{V}_\sigma = 0$ and hence the HJB equations imply the following equations for the value functions and prices:

$$V_{ln} = \frac{\lambda_u V_{hn}}{r+\lambda_u} \tag{26}$$

$$V_{hn} = \frac{\lambda_d V_{ln} + \rho V_{ho} - \rho A}{r+\lambda_d+\rho} \tag{27}$$

$$V_{lo} = \frac{\lambda_u V_{ho} + \rho B + \lambda_T F + \lambda_D(1-f)F + (\rho+\lambda_T+\lambda_D)V_{ln} + C - \delta}{r+\lambda_u+\rho+\lambda_T+\lambda_D} \tag{28}$$

$$V_{ho} = \frac{\lambda_d V_{lo} + (\lambda_T+\lambda_D)V_{hn} + \lambda_T F + \lambda_D(1-f)F + C}{r+\lambda_d+\lambda_T+\lambda_D} \tag{29}$$

Bilateral bargaining between investors and market makers determines bid and ask prices. A low-type owner demands at least $\Delta V_l = V_{lo} - V_{ln}$ for the bond while a high-type nonowner will not pay more than $\Delta V_h = V_{ho} - V_{hn}$. Nash bargaining between investors and market makers in which the outside option of the market maker is to trade in the inter-dealer market results in the bid and ask prices

$$
\begin{aligned}
A &= \Delta V_h z + M(1-z) \\
B &= \Delta V_l z + M(1-z)
\end{aligned}
$$

[13] According to Leibnitz's rule $\frac{\partial}{\partial\alpha}\int_\alpha^\infty F(x,\alpha)dx = \int_\alpha^\infty \frac{\partial F}{\partial \alpha}dx - F(\alpha,\alpha)$.

where $z$ is the bargaining power of the market maker. Thus,

$$\begin{aligned}
(r+\lambda_u)V_{ln} &= \lambda_u V_{hn} \\
(r+\lambda_d)V_{hn} &= \lambda_d V_{ln} + \rho\Delta V_h - \rho[\Delta V_h z + M(1-z)] \\
(r+\lambda_u)V_{lo} &= \lambda_u V_{ho} + \rho[\Delta V_l z + M(1-z)] + \lambda_T F + \lambda_D(1-f)F \\
&\quad -(\rho+\lambda_T+\lambda_D)\Delta V_l + C - \delta \\
(r+\lambda_d)V_{ho} &= \lambda_d V_{lo} - (\lambda_T+\lambda_D)\Delta V_h + \lambda_T F + \lambda_D(1-f)F + C.
\end{aligned}$$

These equations reduce to

$$\begin{aligned}
(r+\lambda_u+(1-z)\rho+\lambda_T+\lambda_D)\Delta V_l &= \lambda_u\Delta V_h + \rho(1-z)M + \psi_c - \delta \\
(r+\lambda_d+(1-z)\rho+\lambda_T+\lambda_D)\Delta V_h &= \lambda_d\Delta V_l + \rho(1-z)M + \psi_c
\end{aligned}$$

where $\psi_c = \lambda_T F + \lambda_D(1-f)F + C$. This implies that

$$\begin{pmatrix} \Delta V_l \\ \Delta V_h \end{pmatrix} = \frac{\rho(1-z)M+\psi_c}{K}\begin{pmatrix} 1 \\ 1 \end{pmatrix} - \frac{\delta}{K(K+\lambda_d+\lambda_u)}\begin{pmatrix} K+\lambda_d \\ \lambda_d \end{pmatrix} \tag{30}$$

where $K = r + (1-z)\rho + \lambda_T + \lambda_D$. The agent faces a bid-ask spread of

$$z(\Delta V_h - \Delta V_l) = \frac{z\delta}{r+(1-z)\rho+\lambda_d+\lambda_u+\lambda_T+\lambda_D}$$

For agents with different search intensities the same arguments can applied. Next, we now find the range of inter-dealer prices for which all investors trade. Define $\rho_0 = \min_{i=1,\ldots,I} \rho^i$. For all $\rho \geq \rho_0$ the inter-dealer price has to satisfy

$$M \leq \Delta V_h = \frac{\rho(1-z)M+\psi_C}{K} - \frac{\delta\lambda_d}{K(K+\lambda_d+\lambda_u)}$$

according to (30) and rearranging this inequality yields

$$M \leq \frac{\psi_c}{r+\lambda_T+\lambda_D} - \frac{\delta\lambda_d}{(r+\lambda_T+\lambda_D)(r+(1-z)\rho+\lambda_d+\lambda_u+\lambda_T+\lambda_D)}.$$

If this holds for all $\rho \geq \rho_0$ then

$$M \leq \frac{\psi_c}{r+\lambda_T+\lambda_D} - \frac{\delta\lambda_d}{(r+\lambda_T+\lambda_D)(r+(1-z)\rho_0+\lambda_d+\lambda_u+\lambda_T+\lambda_D)}.$$

Likewise we find that

$$M \geq \frac{\psi_c}{r+\lambda_T+\lambda_D} - \frac{\delta(K+\lambda_d)}{(r+\lambda_T+\lambda_D)(K+\lambda_d+\lambda_u)}$$

which holds for all $\rho$ if

$$M \geq \frac{\psi_c}{r+\lambda_T+\lambda_D} - \frac{\delta(r+(1-z)\rho_0+\lambda_d+\lambda_T+\lambda_D)}{(r+\lambda_T+\lambda_D)(r+(1-z)\rho_0+\lambda_d+\lambda_u+\lambda_T+\lambda_D)}.$$

# C Proof of Theorem 2.2

In this section I provide a proof of Theorem 2.2. I first find the length of time with more sellers than buyers following a liquidity shock. I then find bid and ask prices prevailing immediately after a liquidity shock.

If we define $\mu_l = \mu_{lo} + \mu_{ln}$ and $\mu_h = \mu_{ho} + \mu_{hn}$ we have according to equations (1)-(4) that

$$\begin{pmatrix} \dot{\mu_l} \\ \dot{\mu_h} \end{pmatrix} = \begin{bmatrix} -\lambda_u & \lambda_d \\ \lambda_u & -\lambda_d \end{bmatrix} \begin{pmatrix} \mu_l \\ \mu_h \end{pmatrix}.$$

The solution to these ODEs is

$$\begin{pmatrix} \mu_l(t) \\ \mu_h(t) \end{pmatrix} = (1 - e^{-t(\lambda_u+\lambda_d)}) \begin{pmatrix} \frac{\lambda_d}{\lambda_d+\lambda_u} \\ \frac{\lambda_u}{\lambda_d+\lambda_u} \end{pmatrix} + e^{-t(\lambda_u+\lambda_d)} \begin{pmatrix} \mu_l(0) \\ \mu_h(0) \end{pmatrix} \tag{31}$$

This result will be useful in a short moment. We now look at $\mu_{hn} - \mu_{lo}$ since sellers are constrained if this difference is negative. We have that

$$\begin{aligned} \dot{\mu_{hn}} - \dot{\mu_{lo}} &= \rho(\mu_m - \mu_{hn}) + (\lambda_T + \lambda_D)(\mu_{ho} + \mu_{lo}) + \lambda_u(\mu_{ln} + \mu_{lo}) - \lambda_d(\mu_{hn} + \mu_{ho}) \\ &= \rho(\mu_m - \mu_{hn}) - (\lambda_T + \lambda_D)(\mu_{hn} - \mu_{lo}) + (\lambda_T + \lambda_D - \lambda_d - \lambda_u)\mu_h + \lambda_u \end{aligned}$$

If sellers are constrained we have $\mu_m = \mu_{hn}$ and

$$\dot{\mu_{hn}} - \dot{\mu_{lo}} = -(\lambda_T + \lambda_D)(\mu_{hn} - \mu_{lo}) + (\lambda_T + \lambda_D - \lambda_d - \lambda_u)\mu_h + \lambda_u.$$

Assume that $\mu_{hn}(0) - \mu_{lo}(0) < 0$. Using the result in equation (31), the solution for $\mu_{hn}(t) - \mu_{lo}(t)$ for any $t$ where $\mu_{hn} - \mu_{lo}$ has not yet been positive is

$$\begin{aligned} \mu_{hn}(t) - \mu_{lo}(t) &= \frac{\lambda_u}{\lambda_d + \lambda_u} + e^{-(\lambda_u+\lambda_d)t}(\mu_h(0) - \frac{\lambda_u}{\lambda_d + \lambda_u}) \\ &\quad -(\mu_{ho}(0) + \mu_{lo}(0))e^{-(\lambda_T+\lambda_D)t}. \end{aligned} \tag{32}$$

This equation has a unique $t_s$ where $\mu_{hn}(t_s) - \mu_{lo}(t_s) = 0$. This implies that if sellers are constrained at time 0 they become unconstrained at time $t_s$. What remains to show is that they stay unconstrained after time $t_s$. If sellers are not constrained we have $\mu_m = \mu_{lo}$ and

$$\dot{\mu_{hn}} - \dot{\mu_{lo}} = -(\rho + \lambda_T + \lambda_D)(\mu_{hn} - \mu_{lo}) + (\lambda_T + \lambda_D - \lambda_d - \lambda_u)\mu_h + \lambda_u$$

The solution is (assuming that $\mu_{hn}(0) - \mu_{lo}(0) = 0$)

$$\begin{aligned} \mu_{hn}(t) - \mu_{lo}(t) &= \frac{\lambda_u}{\lambda_d + \lambda_u}\frac{\lambda_T + \lambda_D}{\rho + \lambda_T + \lambda_D} \\ &\quad + \frac{\lambda_T + \lambda_D - \lambda_d - \lambda_u}{\rho + \lambda_T + \lambda_D - \lambda_d - \lambda_u}(\mu_h(0) - \frac{\lambda_u}{\lambda_d + \lambda_u})e^{-(\lambda_u+\lambda_d)t} \\ &\quad - \Big[(\mu_h(0) - \frac{\lambda_u}{\lambda_u + \lambda_d})\frac{\lambda_T + \lambda_D - \lambda_d - \lambda_u}{\rho + \lambda_T + \lambda_D - \lambda_d - \lambda_u} + \frac{\lambda_u}{\lambda_d + \lambda_u}\frac{\lambda_T + \lambda_D}{\rho + \lambda_T + \lambda_D}\Big]e^{-(\rho+\lambda_T+\lambda_D)t}. \end{aligned}$$

Assume that $\mu_h(0) = (1-s)\frac{\lambda_u}{\lambda_u+\lambda_d}$. Then we have

$$\begin{aligned}\mu_{hn}(t)-\mu_{lo}(t) &= \frac{\lambda_u}{\lambda_d+\lambda_u}\Bigg[\frac{\lambda_T+\lambda_D}{\rho+\lambda_T+\lambda_D} \\ &\quad -s\frac{\lambda_T+\lambda_D-\lambda_d-\lambda_u}{\rho+\lambda_T+\lambda_D-\lambda_d-\lambda_u}e^{-(\lambda_u+\lambda_d)t} \\ &\quad -\Big[\frac{(1-s)(\lambda_T+\lambda_D)(\rho+\lambda_T+\lambda_D-\lambda_u-\lambda_d)+s\rho(\lambda_T+\lambda_D)}{(\rho+\lambda_T+\lambda_D)(\rho+\lambda_T+\lambda_D-\lambda_u-\lambda_d)}\Big]e^{-(\rho+\lambda_T+\lambda_D)t}\Bigg].\end{aligned}$$

If $\rho+\lambda_T+\lambda_D > \lambda_d+\lambda_u$ we have that $\mu_{hn}(t)-\mu_{lo}(t) \geq 0$ for all $t$ . What I have now shown is that if the fractions of investors are such that sellers are constrained, the sellers are constrained a period of time $t_s$ and unconstrained thereafter. Next, I find $t_s$. The steady state value of $\mu_{hn}-\mu_{lo}$ is $\frac{\lambda_u}{\lambda_d+\lambda_u}\frac{\lambda_T+\lambda_D}{\rho+\lambda_T+\lambda_D}$. This, along with equations (1) and (2) yielding

$$\dot{\mu_{lo}}+\dot{\mu_{ho}} = \rho(\mu_{hn}-\mu_{lo})-(\lambda_T+\lambda_D)(\mu_{ho}+\mu_{lo})$$

gives the steady state value of $\mu_{lo}+\mu_{ho}$ as $\frac{\lambda_u}{\lambda_d+\lambda_u}\frac{\rho}{\rho+\lambda_T+\lambda_D}$. Since the steady state value of $\mu_h$ is $\frac{\lambda_u}{\lambda_d+\lambda_u}$ we have according to (32) that the time, $t$, an investor is constrained after a shock of $s$ solves

$$0 = 1-se^{-(\lambda_u+\lambda_d)t}-\frac{\rho}{\rho+\lambda_T+\lambda_D}e^{-(\lambda_T+\lambda_D)t}. \tag{33}$$

If $s \leq \frac{\lambda_T+\lambda_D}{\rho+\lambda_T+\lambda_D}$ the investor is not constrained at any time after the shock and prices do not change.

Next, I find bid and ask prices immediately after the liquidity shock. Rewriting equations (22)-(25) we have that

$$\begin{aligned}\Delta\dot{V}_h &= (C_1+\lambda_d+\rho)\Delta V_h-\lambda_d\Delta V_l-\psi_C-\rho A \\ \Delta\dot{V}_l &= (C_1+\lambda_u+\rho)\Delta V_l-\lambda_u\Delta V_h-(\psi_C-\delta)-\rho B \\ C_1 &= r+\lambda_D+\lambda_T \\ \psi_C &= \lambda_T F+\lambda_D(1-f)F+C.\end{aligned}$$

If sellers are constrained we have that the interdealer price is $M = \Delta V_l$, so $A = \Delta V_h z + \Delta V_l(1-z)$ and $B = \Delta V_l$. Thus,

$$\begin{aligned}\Delta\dot{V}_h &= (C_1+\lambda_d+(1-z)\rho)\Delta V_h+(-\lambda_d-(1-z)\rho)\Delta V_l-\psi_C \\ \Delta\dot{V}_l &= (C_1+\lambda_u)\Delta V_l-\lambda_u\Delta V_h-(\psi_C-\delta)\end{aligned}$$

which can be rewritten as

$$\begin{pmatrix}\Delta\dot{V}_h \\ \Delta\dot{V}_l\end{pmatrix} = \begin{bmatrix}C_1+\lambda_d+(1-z)\rho & -\lambda_d-(1-z)\rho \\ -\lambda_u & C_1+\lambda_u\end{bmatrix}\begin{pmatrix}\Delta V_h \\ \Delta V_l\end{pmatrix}-\begin{pmatrix}\psi_C \\ \psi_C-\delta\end{pmatrix} \tag{34}$$

The reservation values immediately after the shock is found be solving these ODEs backwards from the steady state reservation values for the period of time sellers are constrained. That is, the reservation values after the shock is the time $t_s$ solution to

$$\begin{aligned}\begin{pmatrix} \Delta\dot{V}_h \\ \Delta\dot{V}_l \end{pmatrix} &= -\begin{bmatrix} C_1+\lambda_d+(1-z)\rho & -\lambda_d-(1-z)\rho \\ -\lambda_u & C_1+\lambda_u \end{bmatrix}\begin{pmatrix} \Delta V_h \\ \Delta V_l \end{pmatrix}+\begin{pmatrix} \psi_C \\ \psi_C-\delta \end{pmatrix} \\ \begin{pmatrix} \Delta V_h(0) \\ \Delta V_l(0) \end{pmatrix} &= \begin{pmatrix} \Delta V_h^{ss} \\ \Delta V_l^{ss} \end{pmatrix}.\end{aligned}$$

The steady state solution of this system is

$$\begin{aligned}\begin{pmatrix} \Delta V_h^{\text{imb}} \\ \Delta V_l^{\text{imb}} \end{pmatrix} &= -\begin{bmatrix} -(C_1+\lambda_d+(1-z)\rho) & \lambda_d+(1-z)\rho \\ \lambda_u & -(C_1+\lambda_u) \end{bmatrix}^{-1}\begin{pmatrix} \psi_C \\ \psi_C-\delta \end{pmatrix} \\ &= \begin{pmatrix} \frac{\psi_C}{C_1}-\delta\left(\frac{\sqrt{\gamma}-\lambda_u}{C_1(\sqrt{\gamma}+C_1)}\right) \\ \frac{\psi_C}{C_1}-\delta\left(\frac{\sqrt{\gamma}-\lambda_u}{C_1(\sqrt{\gamma}+C_1)}\right)-\frac{\delta}{\sqrt{\gamma}+C_1} \end{pmatrix}\end{aligned}$$

where $\sqrt{\gamma}=\lambda_d+\lambda_u+(1-z)\rho$. If sellers were always constrained, the reservation values of investors would be $\begin{pmatrix} \Delta V_h^{\text{imb}} \\ \Delta V_l^{\text{imb}} \end{pmatrix}$. Tedious calculations and the use of Corollary 11.3.3 in Bernstein (2005) show that

$$\begin{pmatrix} \Delta V_h(t) \\ \Delta V_l(t) \end{pmatrix} = e^{-tC_1}\begin{pmatrix} \Delta V_h^{ss} \\ \Delta V_l^{ss} \end{pmatrix}+(1-e^{-tC_1})\begin{pmatrix} \Delta V_h^{\text{imb}} \\ \Delta V_l^{\text{imb}} \end{pmatrix}$$

# D Proof of Theorem 2.3

This section provides a proof of Theorem 2.3. Part 1 of the theorem - that bid-ask spreads are unaffected by a liquidity shock - follows immediately from Theorem 2.1 and 2.2. Consequently, any relation between midprices of investors with different levels of sophistication can be shown by showing the relation for bid prices.

I first assume that $\frac{\lambda_T+\lambda_D}{min\{\rho_i,\rho_j\}+\lambda_T+\lambda_D} < s \leq 1$ and prove the second half of part 2 in the theorem by showing that the bid price of an unsophisticated investor minus that of a sophisticated investor, $B_i(s) - B_j(s)$, is increasing in $s$. I show this by proving that $\frac{\partial B(s)}{\partial\rho\partial s} < 0$. Implicit differentiation in equation (33) yields

$$\frac{\partial t}{\partial s} = \frac{1}{s(\lambda_u+\lambda_d) + \frac{\rho(\lambda_T+\lambda_D)}{\rho+\lambda_T+\lambda_D}e^{(\lambda_u+\lambda_d-\lambda_T-\lambda_D)t}}$$

so we have

$$\frac{\partial B(s)}{\partial s} = (r+\lambda_D+\lambda_T)\frac{\partial t}{\partial s}e^{-t(r+\lambda_D+\lambda_T)}(\Delta V_l^{ss} - \Delta V_l^{imb}).$$

Since

$$\begin{aligned}
\Delta V_l^{imb} - \Delta V_l^{ss} &= \frac{\rho(1-z)}{C(r+\lambda_T+\lambda_D)}\left[\frac{(1-\tilde{q})C_0+\lambda_d}{C_0+\lambda_u+\lambda_d} - \frac{C+\lambda_d}{C+\lambda_u+\lambda_d}\right] \\
C_0 &= r+(1-z)\rho_0+\lambda_T+\lambda_D \\
C &= r+(1-z)\rho+\lambda_T+\lambda_D
\end{aligned}$$

we have that

$$\frac{\partial B(s)}{\partial s} = \frac{\frac{\rho(1-z)}{C}\left[\frac{(1-\tilde{q})C_0+\lambda_d}{C_0+\lambda_u+\lambda_d} - \frac{C+\lambda_d}{C+\lambda_u+\lambda_d}\right]e^{-rt}}{s(\lambda_u+\lambda_d)e^{(\lambda_T+\lambda_D)t} + \frac{\rho(\lambda_T+\lambda_D)}{\rho+\lambda_T+\lambda_D}e^{(\lambda_u+\lambda_d)t}}.$$

Now

$$\begin{aligned}
\frac{\partial t}{\partial\rho} &= \frac{\frac{\lambda_T+\lambda_D}{(\rho+\lambda_T+\lambda_D)^2}e^{-(\lambda_T+\lambda_D)t}}{(\lambda_u+\lambda_d)se^{-(\lambda_u+\lambda_d)t} + \frac{\rho(\lambda_T+\lambda_D)}{\rho+\lambda_T+\lambda_D}e^{-(\lambda_T+\lambda_D)t}} \\
\frac{\partial\left(\frac{\rho(C+\lambda_d)}{C(C+\lambda_u+\lambda_d)}\right)}{\partial\rho} &= \frac{\rho(1-z)C\lambda_u + (r+\lambda_T+\lambda_D)(C+\lambda_d)(C+\lambda_d+\lambda_u)}{C^2(C+\lambda_u+\lambda_d)^2} \\
\frac{\partial\left(\frac{\rho(1-z)}{C}\right)}{\partial\rho} &= \frac{(1-z)(r+\lambda_T+\lambda_D)}{C^2}
\end{aligned}$$

so the numerator in $\frac{\partial B(s)}{\partial\rho\partial s}$ is

$$\Big[\tfrac{(1-z)(r+\lambda_T+\lambda_D)}{C^2}\tfrac{(1-\tilde{q})C_0+\lambda_d}{C_0+\lambda_u+\lambda_d}-(1-z)\tfrac{\rho(1-z)C\lambda_u+(r+\lambda_T+\lambda_D)(C+\lambda_d)(C+\lambda_d+\lambda_u)}{C^2(C+\lambda_u+\lambda_d)^2}\Big]\times$$

$$\Big[s(\lambda_u+\lambda_d)e^{(\lambda_T+\lambda_D)t}+\tfrac{\rho(\lambda_T+\lambda_D)}{\rho+\lambda_T+\lambda_D}e^{(\lambda_u+\lambda_d)t}\Big]-\tfrac{\rho(1-z)}{C}\Big[\tfrac{(1-\tilde{q})C_0+\lambda_d}{C_0+\lambda_u+\lambda_d}-\tfrac{C+\lambda_d}{C+\lambda_u+\lambda_d}\Big]\times$$

$$\Big[(se^{(\lambda_T+\lambda_D)t}+\tfrac{\rho}{\rho+\lambda_T+\lambda_D}e^{(\lambda_d+\lambda_u)t})\tfrac{(\lambda_u+\lambda_d)\frac{(\lambda_T+\lambda_D)^2}{(\rho+\lambda_T+\lambda_D)^2}e^{-(\lambda_D+\lambda_T)t}}{(\lambda_u+\lambda_d)se^{-(\lambda_u+\lambda_d)t}+\frac{\rho(\lambda_T+\lambda_D)}{\rho+\lambda_T+\lambda_D}e^{-(\lambda_T+\lambda_D)t}}+\tfrac{(\lambda_T+\lambda_D)^2}{(\rho+\lambda_T+\lambda_D)^2}e^{(\lambda_u+\lambda_d)t}\Big]=$$

$$\left[\tfrac{(1-\tilde{q})C_0+\lambda_d}{C_0+\lambda_u+\lambda_d}-\tfrac{C+\lambda_d}{C+\lambda_u+\lambda_d}\right]\tfrac{1-z}{C}\left[\tfrac{r+\lambda_T+\lambda_D}{C}\Big[s(\lambda_u+\lambda_d)e^{(\lambda_T+\lambda_D)t}+\tfrac{\rho(\lambda_T+\lambda_D)}{\rho+\lambda_T+\lambda_D}e^{(\lambda_u+\lambda_d)t}\Big]-\right.$$

$$\left.\rho\Big[(se^{(\lambda_T+\lambda_D)t}+\tfrac{\rho}{\rho+\lambda_T+\lambda_D}e^{(\lambda_d+\lambda_u)t})\tfrac{(\lambda_u+\lambda_d)\frac{(\lambda_T+\lambda_D)^2}{(\rho+\lambda_T+\lambda_D)^2}e^{-(\lambda_D+\lambda_T)t}}{(\lambda_u+\lambda_d)se^{-(\lambda_u+\lambda_d)t}+\frac{\rho(\lambda_T+\lambda_D)}{\rho+\lambda_T+\lambda_D}e^{-(\lambda_T+\lambda_D)t}}+\tfrac{(\lambda_T+\lambda_D)^2}{(\rho+\lambda_T+\lambda_D)^2}e^{(\lambda_u+\lambda_d)t}\Big]\right]$$

$$-\tfrac{(1-z)^2\rho\lambda_u}{C(C+\lambda_u+\lambda_d)^2}\Big[s(\lambda_u+\lambda_d)e^{(\lambda_T+\lambda_D)t}+\tfrac{\rho(\lambda_T+\lambda_D)}{\rho+\lambda_T+\lambda_D}e^{(\lambda_u+\lambda_d)t}\Big]=$$

$$\left[\tfrac{(1-\tilde{q})C_0+\lambda_d}{C_0+\lambda_u+\lambda_d}-\tfrac{C+\lambda_d}{C+\lambda_u+\lambda_d}\right]\tfrac{1-z}{C}\times$$

$$\left[\Big(\tfrac{r+\lambda_T+\lambda_D}{C}+\tfrac{(1-z)\rho\lambda_u}{(C+\lambda_u+\lambda_d)^2}\big(\tfrac{C+\lambda_d}{C+\lambda_u+\lambda_d}-\tfrac{(1-\tilde{q})C_0+\lambda_d}{C_0+\lambda_u+\lambda_d}\big)^{-1}\Big)\Big[s(\lambda_u+\lambda_d)e^{(\lambda_T+\lambda_D)t}+\tfrac{\rho(\lambda_T+\lambda_D)}{\rho+\lambda_T+\lambda_D}e^{(\lambda_u+\lambda_d)t}\Big]-\right.$$

$$\left.\rho\Big[(se^{(\lambda_T+\lambda_D)t}+\tfrac{\rho}{\rho+\lambda_T+\lambda_D}e^{(\lambda_d+\lambda_u)t})\tfrac{(\lambda_u+\lambda_d)\frac{(\lambda_T+\lambda_D)^2}{(\rho+\lambda_T+\lambda_D)^2}e^{-(\lambda_D+\lambda_T)t}}{(\lambda_u+\lambda_d)se^{-(\lambda_u+\lambda_d)t}+\frac{\rho(\lambda_T+\lambda_D)}{\rho+\lambda_T+\lambda_D}e^{-(\lambda_T+\lambda_D)t}}+\tfrac{(\lambda_T+\lambda_D)^2}{(\rho+\lambda_T+\lambda_D)^2}e^{(\lambda_u+\lambda_d)t}\Big]\right]\leq$$

$$\left[\tfrac{(1-\tilde{q})C_0+\lambda_d}{C_0+\lambda_u+\lambda_d}-\tfrac{C+\lambda_d}{C+\lambda_u+\lambda_d}\right]\tfrac{1-z}{C}\times$$

$$\left[s(\lambda_u+\lambda_d)e^{(\lambda_T+\lambda_D)t}\big(\tfrac{r+\lambda_T+\lambda_D}{C}+\tfrac{(1-z)\rho\lambda_u}{(C+\lambda_u+\lambda_d)^2}-\tfrac{\lambda_T+\lambda_D}{\rho+\lambda_T+\lambda_D}\big)+\right.$$

$$\left.\tfrac{\rho(\lambda_T+\lambda_D)}{\rho+\lambda_T+\lambda_D}e^{(\lambda_u+\lambda_d)t}\big(\tfrac{r+\lambda_T+\lambda_D}{C}+\tfrac{(1-z)\rho\lambda_u}{(C+\lambda_u+\lambda_d)^2}-\tfrac{\lambda_T+\lambda_D}{\rho+\lambda_T+\lambda_D}-\tfrac{\rho}{\rho+\lambda_T+\lambda_D}\tfrac{(\lambda_u+\lambda_d)\frac{\lambda_T+\lambda_D}{\rho+\lambda_T+\lambda_D}e^{-(\lambda_D+\lambda_T)t}}{(\lambda_u+\lambda_d)se^{-(\lambda_u+\lambda_d)t}+\frac{\rho(\lambda_T+\lambda_D)}{\rho+\lambda_T+\lambda_D}e^{-(\lambda_T+\lambda_D)t}}\big)\right].$$

We have that $\frac{(1-\tilde{q})C_0+\lambda_d}{C_0+\lambda_u+\lambda_d}-\frac{C+\lambda_d}{C+\lambda_u+\lambda_d}<0$ and $\frac{r+\lambda_T+\lambda_D}{C}>\frac{\lambda_T+\lambda_D}{\rho+\lambda_T+\lambda_D}$. Also

$$\frac{(1-z)\rho\lambda_u}{(C+\lambda_u+\lambda_d)^2}-\frac{\rho}{\rho+\lambda_T+\lambda_D}\frac{(\lambda_u+\lambda_d)\frac{\lambda_T+\lambda_D}{\rho+\lambda_T+\lambda_D}e^{-(\lambda_D+\lambda_T)t}}{(\lambda_u+\lambda_d)se^{-(\lambda_u+\lambda_d)t}+\frac{\rho(\lambda_T+\lambda_D)}{\rho+\lambda_T+\lambda_D}e^{-(\lambda_T+\lambda_D)t}}$$
$$=\frac{(1-z)\rho\lambda_u}{(C+\lambda_u+\lambda_d)^2}-\frac{1}{\frac{(\rho+\lambda_T+\lambda_D)^2}{(\lambda_T+\lambda_D)\rho}se^{(\lambda_T+\lambda_D-\lambda_u-\lambda_d)t}+\frac{\rho+\lambda_T+\lambda_D}{\lambda_u+\lambda_d}}.$$

A sufficient condition for $\frac{\partial B}{\partial\rho\partial s}<0$ is that $\frac{(1-z)\rho\lambda_u}{(C+\lambda_u+\lambda_d)^2}>\frac{1}{\frac{(\rho+\lambda_T+\lambda_D)^2}{(\lambda_T+\lambda_D)\rho}se^{(\lambda_T+\lambda_D-\lambda_u-\lambda_d)t}+\frac{\rho+\lambda_T+\lambda_D}{\lambda_u+\lambda_d}}$. This is the case if $f(s)>0$ for every $0\leq s\leq 1$ where $f$ is defined as

$$f(s)=\frac{(1-z)\rho\lambda_u}{(C+\lambda_u+\lambda_d)^2}\Big[\frac{(\rho+\lambda_T+\lambda_D)^2}{(\lambda_T+\lambda_D)\rho}se^{(\lambda_T+\lambda_D-\lambda_u-\lambda_d)t}+\frac{\rho+\lambda_T+\lambda_D}{\lambda_u+\lambda_d}\Big]-1.$$

Since $f(s)>f(0)$ for $s>0$ we need to show that $f(0)>0$. We have that

$$\begin{aligned}(1-z)\rho\lambda_u(\rho+\lambda_T+\lambda_D)-(C+\lambda_u+\lambda_d)^2(\lambda_u+\lambda_d)=&\\ \big[(1-z)(z\lambda_u-\lambda_d(1-z))\big]\rho^2-&\\ (1-z)\big[(2\lambda_d+\lambda_u)(\lambda_T+\lambda_D)+2(\lambda_u+\lambda_d)^2\big]\rho-&\\ (\lambda_u+\lambda_d)(\lambda_T+\lambda_D+\lambda_u+\lambda_d)^2.&\end{aligned}$$

The last expression is a second-order polynomium in $\rho$ and if this polynomium is larger than 0, then so is f(0). If $(1-z)(z\lambda_u - \lambda_d(1-z)) > 0$ then the polynomium is positive for $\rho$ sufficiently large. If $\lambda_u > \frac{1-z}{z}\lambda_d$ then $(1-z)(z\lambda_u - \lambda_d(1-z)) > 0$. Thus, if $\rho$ is sufficiently large and $\lambda_u > \frac{1-z}{z}\lambda_d$ then $\frac{\partial B(s)}{\partial\rho\partial s} < 0$ as needed to be proven.

# E Additional details on estimation methodology

## E.1 Squared errors in likelihood estimation

In the text $A_{tbi}$ is defined as the $i$'th ask yield on bond $b$ and day $t$ and $B_{tbi}$ as the corresponding bid yield. Fitted yields follow the same notation with an addition superscript $M$. The sum of squared errors in the likelihood in equation (14) equal

$$\sum_{t=1}^{T}\sum_{b=1}^{N_b} w_{tb} \sum_{i=1}^{N_{tb}} \Big[[(A_{tbi} - \overline{AB}_{tb}) - (A_{tbi}^M - \overline{AB}_{tb}^M)]^2 + [(B_{tbi} - \overline{AB}_{tb}) - (B_{tbi}^M - \overline{AB}_{tb}^M)]^2\Big]$$

where I use the notation

$$\begin{aligned}
\overline{A}_{tb} &= \frac{1}{N_{tb}}\sum_{m=1}^{N_{tb}} A_{tbm} \\
\overline{B}_{tb} &= \frac{1}{N_{tb}}\sum_{m=1}^{N_{tb}} B_{tbm} \\
\overline{AB}_{tb} &= \frac{1}{2}(\overline{A}_{tb} + \overline{B}_{tb}).
\end{aligned}$$

We have that

$$\begin{aligned}
&\sum_{i=1}^{N_{tb}}[(A_{tbi} - \overline{AB}_{tb}) - (A_{tbi}^M - \overline{AB}_{tb}^M)]^2 \\
&= \sum_{i=1}^{N_{tb}}[(A_{tbi} - A_{tbi}^M) - (\overline{A}_{tb} - \overline{A}_{tb}^M)]^2 + N_{tb}\Big[(\overline{A}_{tb} - \overline{A}_{tb}^M) - (\overline{AB}_{tb} - \overline{AB}_{tb}^M)\Big]^2
\end{aligned}$$

and

$$\begin{aligned}
&\sum_{i=1}^{N_{tb}}[(B_{tbi} - \overline{AB}_{tb}) - (B_{tbi}^M - \overline{AB}_{tb}^M)]^2 \\
&= \sum_{i=1}^{N_{tb}}[(B_{tbi} - B_{tbi}^M) - (\overline{B}_{tb} - \overline{B}_{tb}^M)]^2 + N_{tb}\Big[(\overline{B}_{tb} - \overline{B}_{tb}^M) - (\overline{AB}_{tb} - \overline{AB}_{tb}^M)\Big]^2.
\end{aligned}$$

Since

$$\begin{aligned}
&N_{tb}\Big[(\overline{A}_{tb} - \overline{A}_{tb}^M) - (\overline{AB}_{tb} - \overline{AB}_{tb}^M)\Big]^2 + N_{tb}\Big[(\overline{B}_{tb} - \overline{B}_{tb}^M) - (\overline{AB}_{tb} - \overline{AB}_{tb}^M)\Big]^2 \\
&= \frac{N_{tb}}{2}\Big[(\overline{A}_{tb} - \overline{B}_{tb}) - (\overline{A}_{tb}^M - \overline{B}_{tb}^M)\Big]^2
\end{aligned}$$

the squared errors equal

$$\sum_{t=1}^{T}\sum_{b=1}^{N_b} w_{tb}\Big[\sum_{i=1}^{N_{tb}}[(A_{tbi}-\overline{A}_{tb})-(A_{tbi}^M-\overline{A}_{tb}^M)]^2+\sum_{i=1}^{N_{tb}}[(B_{tbi}-\overline{B}_{tb})-(B_{tbi}^M-\overline{B}_{tb}^M)]^2$$
$$+\frac{N_{tb}}{2}[(\overline{A}_{tb}-\overline{B}_{tb})-(\overline{A}_{tb}^M-\overline{B}_{tb}^M)]^2\Big].$$

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